PHYSICS
KAS -101/201
(SEMESTER-I/II)

Notes For-B. Tech. First Year

Course coordinator
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PHYSICS

Module - 1 Relativistic Mechanics: [8]

Frame of reference, Inertial & non-inertial frames, Galilean transformations, Michelson-Morley experiment, Postulates of special theory of relativity, Lorentz transformations, Length contraction, Time dilation, Velocity addition theorem, Variation of mass with velocity, Einstein’s mass energy relation, Relativistic relation between energy and momentum, Massless particle.

Module- 2 Electromagnetic Field Theory: [8]

Continuity equation for current density, Displacement current, Modifying equation for the curl of magnetic field to satisfy continuity equation, Maxwell’s equations in vacuum and in non-conducting medium, Energy in an electromagnetic field, Poynting vector and Poynting theorem, Plane electromagnetic waves in vacuum and their transverse nature. Relation between electric and magnetic fields of an electromagnetic wave, Energy and momentum carried by electromagnetic waves, Resultant pressure, Skin depth.

Module- 3 Quantum Mechanics: [8]

Black body radiation, Stefan’s law, Wien’s law, Rayleigh-Jeans law and Planck’s law, Wave particle duality, Matter waves, Time-dependent and time-independent Schrödinger wave equation, Born interpretation of wave function, Solution to stationary state Schrödinger wave equation for one-Dimensional particle in a box, Compton effect.

Module- 4 Wave Optics: [10]

Coherent sources, Interference in uniform and wedge shaped thin films, Necessity of extended sources, Newton’s Rings and its applications. Fraunhofer diffraction at single slit and at double slit, absent spectra, Diffraction grating, Spectra with grating, Dispersive power, Resolving power of grating, Rayleigh’s criterion of resolution, Resolving power of grating.

Module- 5 Fibre Optics & Laser: [10]


Course Outcomes:
1. To solve the classical and wave mechanics problems
2. To develop the understanding of laws of thermodynamics and their application in various processes
3. To formulate and solve the engineering problems on Electromagnetism & Electromagnetic Field Theory
4. To aware of limits of classical physics & to apply the ideas in solving the problems in their parent streams

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Inertial and Non Inertial Frames:

Frames of reference are the coordinate system used to describe the motion of a body.

There are two types of frames of reference.

1. Inertial or non-accelerating frames of reference
2. Non-inertial or accelerating frames of reference

The inertial frame obeys the Newton's law of inertia and the non-inertial frame does not obey Newton's low of inertia. Earth is non-inertial frame of reference, because it has acceleration due to spin motion about its axis and orbital motion around the sun.

Galilean Transformations:

The Galilean transformations equations are used to transform the coordinates of position and time from one inertial frame to the other. The equations relating the coordinates of a particle in two inertial frames are called as Galilean transformations. Consider the two inertial frames of reference F and F'. Let the frame F' is moving with constant velocity v with reference to frame F. The frames F and F' are shown in Fig.1

Let some event occurs at the point P at any instant of time t. The coordinates of point P with respect to frame F are x, y, z, t and with respect to frame F' are x', y', z', t'. Let at t = t'= 0, the origin O of frame F and O' of frame F' coincides with one another. Also axes x and x' are parallel to v. Let y' and z' are parallel to y and z respectively.

From Fig.

\[ x = x' + vt \]  \[ \text{...........................................}(1) \]

\[ x' = x - vt \]  \[ \text{...........................................}(2) \]

As there is no relative motion along y and z- axes, we can writ

\[ y' = y \]  \[ \text{.........(3)}, \]

\[ z' = z \]  \[ \text{.........(4), and t'=t.........(5)} \]

These equations are called as Galilean transformation equations. The inverse Galilean transformation can be written as,

\[ x = x' + vt, \quad y = y', \quad z = z \quad \text{and} \quad t = t' \]

Transformation in velocities components:

The conversion of velocity components measured in frame F into their equivalent components in the frame F' can be known by differential Equation (1) with respect to time we get,
\[ u'_x = \frac{dx}{dt} = \frac{dx}{dt} (x - vt) = \frac{dx}{dt} - \frac{d}{dt} (x - vt) \]

\[ \text{hence } u'_x = u_x - v \]

Similarly, from Equation (3) and (4) we can write

\[ u'_y = u_y \quad \text{and} \quad u'_z = u_z \]

In vector form,

\[ \mathbf{u}' = \mathbf{u} - \mathbf{v} \]

**Transformation in acceleration components:** The acceleration components can be derived by differentiating velocity equations with respect to time,

\[ a'_x = \frac{du'_x}{dt} = \frac{d}{dt} (u_x - v) \]

\[ a'_x = a_x \]

In vector form \[ \mathbf{a}' = \mathbf{a} \]

This shows that in all inertial reference frames a body will be observed to have the same acceleration. Hence acceleration components are invariant.

**Failures of Galilean transformation:**

1. According to Galilean transformations the laws of mechanics are invariant. But under Galilean transformations, the fundamental equations of electricity and magnetism have very different forms.

2. Also if we measure the speed of light \( c \) along x-direction in the frame \( F \) and then in the frame \( F' \) the value comes to be \( c' = c - vx \). But according to special theory of relativity the speed of light \( c \) is same in all inertial frames.

**Michelson-Morley Experiment**

“The objective of Michelson - Morley experiment was to detect the existence of stationary medium ether (stationary frame of reference i.e. ether frame.)”, which was assumed to be required for the propagation of the light in the space.

In order to detect the change in velocity of light due to relative motion between earth and hypothetical medium ether, Michelson and Morley performed an experiment which is discussed below : The experimental arrangement is shown in Fig
Light from a monochromatic source S, falls on the semi-silvered glass plate G inclined at an angle 45° to the beam. It is divided into two parts by the semi silvered surface, one ray 1 which travels towards mirror M₁ and other is transmitted, ray 2 towards mirror M₂. These two rays fall normally on mirrors M₁ and M₂ respectively and are reflected back along their original paths and meet at point G and enter in telescope. In telescope interference pattern is obtained.

If the apparatus is at rest in ether, the two reflected rays would take equal time to return the glass plate G. But actually the whole apparatus is moving along with the earth with a velocity say v. Due to motion of earth the optical path traversed by both the rays are not the same. Thus the time taken by the two rays to travel to the mirrors and back to G will be different in this case.

Let the mirrors M₁ and M₂ are at equal distance l from the glass plate G. Further let c and v be the velocities at light and apparatus or earth respectively. It is clear from Fig. that the reflected ray 1 from glass plate G strikes the mirror M₁ at A' and not at A due to the motion of the earth.

The total path of the ray from G to A' and back will be GA'G'.

\[ (c \, t)^2 = (v \, t)^2 + (l)^2 \]

Hence 

\[ t = \frac{1}{\sqrt{c^2 - v^2}} \]

If \( t₁ \) be the time taken by the ray to travel the whole path GA'G', then

\[ t₁ = 2t = \frac{2l}{\sqrt{c^2 - v^2}} = \frac{2l}{c} \left(1 - \frac{v^2}{c^2}\right)^{-1/2} \]
\[ t_1 = \frac{2l}{c} \left( 1 + \frac{v^2}{2c^2} \right) \] ………………(2) Using Binomial Theorem

Now, in case of transmitted ray 2 which is moving longitudinally towards mirror M2, it has a velocity \((c - v)\) relative to the apparatus when it is moving from G to B. During its return journey, its velocity relative to apparatus is \((c + v)\). If \(t_2\) be the total time taken by the longitudinal ray to reach G', then

\[ T_2 = \frac{1}{c-v} + \frac{1}{c+v} \text{ after solving} \]

\[ t_2 = \frac{2l}{c} \left( 1 + \frac{v^2}{c^2} \right) \] ………………(3)

Thus, the difference in times of travel of longitudinal and transverse journeys is

\[ \Delta t = t_2 - t_1 = \frac{2l}{c} \left( 1 + \frac{v^2}{c^2} \right) - \frac{2l}{c} \left( 1 + \frac{v^2}{2c^2} \right) \]

\[ \Delta t = \frac{1v^2}{c^3} \] ………………(4)

The optical path difference between two rays is given as,

\[ \text{Optical path difference} (\Delta) = \text{Velocity} \times t = c \times \Delta t \]

\[ = c \times = \frac{1v^2}{c^3} \]

\[ \Delta = \frac{1v^2}{c^3} \] ………………(5)

If \(\lambda\) is the wavelength of light used, then path difference in terms of wavelength is, \(\frac{1v^2}{\lambda c^3}\).

Michelson-Morley perform the experiment in two steps. First by setting as shown in fig and secondly by turning the apparatus through 90°. Now the path difference is in opposite direction i.e. path difference is \(-\frac{1v^2}{\lambda c^3}\).

Hence total fringe shift \[\Delta N = \frac{2lv^2}{\lambda c^3}\]

Michelson and Morley using \(l=11\) m, \(\lambda= 5800 \times 10^{-10}\) m, \(v= 3 \times 10^4\) m/sec and \(c= 3 \times 10^8\) m/sec

\[ \therefore \text{ Change in fringe shift } \Delta N = \frac{2lv^2}{\lambda c^3} \text{ substitute all these values} \]

\[ = 0.37 \text{ fringe} \]

But the experimental were detect no fringe shift. So there was some problem in theory calculation and is a negative result. The conclusion drawn from the Michelson-Morley experiment is that, there is no existence of stationary medium ether in space.
Negative results of Michelson - Morley experiment:

1. **Ether drag hypothesis:** In Michelson - Morley experiment it is explained that there is no relative motion between the ether and earth. Whereas the moving earth drags ether alone with its motion so the relative velocity of ether and earth will be zero.

2. **Lorentz-Fitzgerald Hypothesis:** Lorentz told that the length of the arm (distance between the pale and the mirror M₂) towards the transmitted side should be \( L(\sqrt{1 - v^2/c^2}) \) but not \( L \). If this is taken then theory and experimental will get matched. But this hypothesis is discarded as there was no proof for this.

3. **Constancy of Velocity of light:** In Michelson - Morley experiment the null shift in fringes was observed. According to Einstein the velocity of light is constant it is independent of frame of reference, source and observer.

**Einstein special theory of Relativity (STR):**

Einstein gave his special theory of relativity (STR) on the basis of M-M experiment

1. **Einstein’s First Postulate of theory of relativity:**
   
   All the laws of physics are same (or have the same form) in all the inertial frames of reference moving with uniform velocity with respect to each other. (This postulate is also called the law of equivalence).

2. **Einstein’s second Postulate of theory of relativity:**
   
   The speed of light is constant in free space or in vacuum in all the inertial frames of reference moving with uniform velocity with respect to each other. (This postulate is also called the law of constancy).

**Lorentz Transformation Equations**

Consider the two observers O and O’ at the origin of the inertial frame of reference F and F’ respectively as shown in Fig. Let at time \( t = t' = 0 \), the two coordinate systems coincide initially. Let a pulse of light is flashed at time \( t = 0 \) from the origin which spreads out in the space and at the same time the frame F’ starts moving with constant velocity \( v \) along positive X-direction relative to the frame F. This pulse of light reaches at point P, whose coordinates of position and time are \((x, y, z, t)\) and \((x', y', z', t')\) measured by the observer O and O’ respectively. Therefore the transformation equations of \( x \) and \( x' \) can be given as,

\[
x' = k (x - vt) \tag{1}
\]

Where \( k \) is the proportionality constant and is independent of \( x \) and \( t \).

The inverse relation can be given as, \( x = k \left( x' + vt' \right) \) \tag{2}

As \( t \) and \( t' \) are not equal, substitute the value of \( x' \) from Equation (1) in Equation (2)

\[
x = k \left[ k (x - vt) + vt' \right] \quad \text{or} \quad \frac{x}{k} = (kx - kvt + vt')
\]
t' = \frac{x}{kv} - \frac{kx}{v} + kt \quad \text{or} \quad t' = kt - \frac{kx}{v} \left(1 - \frac{1}{k^2}\right) \quad \text{.................................(3)}

According to second postulate of special theory of relativity the speed of light $c$ remains constant. Therefore the velocity of pulse of light which spreads out from the common origin observed by observer $O$ and $O'$ should be same.

$\therefore \quad x = ct \quad \text{and} \quad x' = ct' \quad \text{........................................(4)}$

Substitute the values of $x$ and $x'$ from Equation (4) in Equation (1) and (2) we get

$ct' = k(x - v t) = k(ct - vt) \quad \text{or} \quad ct' = kt(c - v) \quad \text{.................................(5)}$

and similarly

$ct = k(t'c + v) \quad \text{.................................(6)}$

Multiplying Equation (5) and (6) we get,

$c^2 t' = k^2 t' \left(c^2 - v^2\right)$ hence

after solving

$k = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{.................................(7)}$

Hence equation (7) substitute in equation (1), then Lorentz transformation in position will be

\[
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
y' &= y \\
z' &= z
\end{aligned}
\]

Calculation of Time: equation (7) substitute in equation (3),

$t' = kt - \frac{kx}{v} \left(1 - \frac{1}{k^2}\right)$

From equation (7),

$\frac{1}{k^2} = 1 - \frac{v^2}{c^2}$

$\frac{v^2}{c^2} = 1 - \frac{1}{k^2}$ then above equation will be

$t' = kt - \frac{kxv^2}{c^2} = kt - \frac{kxv}{c^2}$

Then $t' = k \left(t - \frac{xv}{c^2}\right)$ hence

$t' = \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}$

Hence the Lorentz transformation equations becomes,

\[
\begin{aligned}
x' &= \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \\
y &= y' \\
z &= z \quad \text{and} \\
t' &= \frac{t - \frac{xv}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}}
\end{aligned}
\]

Under the condition $v \ll c$ Lorentz transformation equation can be converted in to Galilean Transformation
Applications of Lorentz Transformation

1. Length contraction
2. Time dilation
3. Relativistic addition of velocity

Length contraction:

Consider a rod at rest in a moving frame of reference $F'$ moving along $x$-direction with constant velocity $v$, relative to the fixed frame of reference $F$ as shown in Fig.

The observer in the frame $F'$ measures the length of rod $AB$ at any instant of time $t$. This length $L_o$ measured in the system in which the rod is at rest is called proper length, therefore $L_o$ is given as,

$$L_o = x_2' - x_1' \quad \text{…………………(1)}$$

Where $x_1'$ and $x_2'$ are the coordinates of the two ends of the rod at any instant. At the same time, the length of the rod is measured by an observer $O$ in his frame say $L$, then

$$L = x_2 - x_1 \quad \text{…………………(2)}$$

Where $x_1$ and $x_2$ are the coordinates of the rod $AB$ respectively with respect to the frame $F$. According to Lorentz transformation equation

$$x_1' = \frac{x_1 - vt}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{and} \quad x_2' = \frac{x_2 - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence

$$x_2' - x_1' = \frac{x_2 - x_1}{\sqrt{1 - \frac{v^2}{c^2}}}$$

By using Equations (1) and (2) we can write

$$L_o = \frac{L}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Hence

$$L = L_o \sqrt{1 - \frac{v^2}{c^2}} \quad \text{…………………(3)}$$

From this equation $L << L_o$. Thus the length of the rod is contracted by a factor $\sqrt{1 - \frac{v^2}{c^2}}$ as measured by observer in stationary frame $F$.

Special Case:
If \( v << c \), then \( v^2/c^2 \) will be negligible in \( L_0 \sqrt{1 - \frac{v^2}{c^2}} \) and it can be neglected.

Then equation (3) becomes \( L = L_0 \).

Percentage of length contraction = \( \frac{L_0 - L}{L_0} \times 100 \)

**Time dilation**

Let there are two inertial frames of references \( F \) and \( F' \). \( F \) is the stationary frame of reference and \( F' \) is the moving frame of reference. At time \( t=t'=0 \) that is in the start, they are at the same position that is Observers \( O \) and \( O' \) coincides. After that \( F' \) frame starts moving with a uniform velocity \( v \) along \( x \) axis.

Let a clock is placed in the frame \( F' \). The time coordinate of the initial time of the clock will be \( t_1 \) according to the observer in \( S \) and the time coordinate of the final tick (time ) will be will be \( t_2 \) according to same observer.

The time coordinate of the initial time of the clock will be \( t'_1 \) according to the observer in \( F' \) and the time coordinate of the final tick (time ) will be will be \( t'_2 \) according to same observer.

Therefore the time of the object as seen by observer \( O' \) in \( F' \) at the position \( x' \) will be

\[
 t_0 = t'_2 - t'_1 \quad \text{------------------------------(1)}
\]

The time \( t' \) is called the proper time of the event.

The apparent or dilated time of the same event from frame \( S \) at the same position \( x \) will be

\[
 t = t_2 - t_1 \quad \text{------------------------------(2)}
\]

Now use Lorentz inverse transformation equations for, that is

\[
 t_1 = \frac{t'_1 + \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{------------------------------- (3)}
\]

\[
 t_2 = \frac{t'_2 + \frac{vx}{c^2}}{\sqrt{1 - \frac{v^2}{c^2}}} \quad \text{------------------------------- (3)}
\]

By putting equations (3) and (4) in equation (2) and solving, we get

\[
 t = \frac{t'_2 - t'_1}{\sqrt{1 - \frac{v^2}{c^2}}}
\]

Substitute equation (1) in above equation,

\[
 t = \frac{t_0}{\sqrt{1 - \frac{v^2}{c^2}}}
\]
This is the relation of the time dilation.

**Special Case:**

If $v << c$, then $v^2/c^2$ will be negligible in $\frac{t_0}{\sqrt{1-v^2/c^2}}$ and it can be neglected.

Then $t = t_0$.

**Experimental evidence:** The time dilation is real effect can be verified by the following experiment. In 1971 NASA conducted one experiment in which J.C. Hafele, as astronomer and R.F. Keating, a physicist circled the earth twice in a jet plane, once from east to west for two days and then from west to east for two days carrying two cesium-beam atomic clocks capable of measuring time to a nanosecond. After the trip the clocks were compared with identical clocks. The clocks on the plane lost $59 \pm 10$ ns during their eastward trip and gained $273 \pm 7$ ns during the westward trip. This results shows that time dilation is real effect.

**Relativistic Addition of Velocities**

One of the consequences of the Lorentz transformation equations is the counter-intuitive “velocity addition theorem”. Consider an inertial frame $S$ moving with uniform velocity $v$ relative to stationary observer $S$ along the positive direction of $X$-axis. Suppose a particle is also moving along the positive direction of $X$-axis. If the particle moves through a distance $dx$ in time interval $dt$ in frame $S$, then velocity of the particle as measured by an observer in this frame is given by

$$u = \frac{dx}{dt} \quad (1)$$

To an observer in $S'$ frame, let the velocity be (by definition)

$$u' = \frac{dx'}{dt'} \quad (2)$$

Now, we have the Lorentz transformation equations:

$$x' = \frac{x-vt}{\sqrt{1-v^2/c^2}} \quad \text{and} \quad t' = \frac{t-vx}{\sqrt{1-v^2/c^2}} \quad (3)$$

Taking differentials of above equations, we get

$$dx' = \frac{dx-vdt}{dt-vdx/c^2} \quad \text{and} \quad dt' = \frac{dt-vdx/c^2}{\sqrt{1-v^2/c^2}} \quad (4)$$

Using eq (4) in eq.(2), we get

$$u' = \frac{dx-vdt}{dt-vdx/c^2} = \frac{\left(\frac{dx}{dt}\right) - v}{(1-vdx/c^2 dt)} \quad (5)$$

Or,
This is the relativistic velocity addition formula. If the speeds \( u \) and \( v \) are small compared to the speed of light, above formula reduces to the Newtonian velocity addition formula:

\[
u' = u - v
\]

Inverse of the formula (6) enables us to find velocity of a particle in \( S \) frame if it is given in \( S' \) frame:

\[
u = \frac{\nu' + v}{1 + \nu'v/c^2}
\]

**Variation of Mass with Velocity**

In Newtonian physics mass of an object used to be an absolute entity, same in all frames. One of the major unusual consequences of relativity was relativity of mass. In the framework of relativity, it can be shown that mass of an object increases with its velocity. We have the following equation expressing the variation of mass with velocity:

\[
m = \frac{m_0}{\sqrt{1 - \nu^2/c^2}}
\]

Where, \( m_0 \) is the mass of the object at rest, known as *rest mass* and \( \nu \) is its velocity relative to observer. As it is clear from above equation, if \( \nu \ll c \), \( m_0 \cong m \), in agreement with common experience.

**Derivation:**

We use relativistic law of momentum conservation to arrive at eq (1). Consider two inertial frames \( S \) and \( S' \), \( S' \) moving with respect to \( S \) with velocity \( v \) along positive X-axis. Let two masses \( m_1 \) and \( m_2 \) are moving with velocities \( u' \) and \(-u'\) with respect to moving frame \( S' \).

Now, let us analyse the collision between two bodies with respect to frame \( S \). If \( u_1 \) and \( u_2 \) are velocities of two masses with respect to frame \( S \), then from velocity addition theorem,

\[
u_1 = \frac{u' + v}{1 + u'v/c^2} \quad \text{and} \quad \nu_2 = \frac{-u' + v}{1 - u'v/c^2}
\]

At the time of collision two masses are momentarily at rest relative to frame \( S' \), but as seen from frame \( S \) they are still moving with velocity \( v \). Since we assume momentum to be conserved even in relativity theory, as seen from \( S \) frame,

\[
\text{Momentum before collision} = \text{momentum after collision}
\]

\[
m_1 \nu_1 + m_2 \nu_2 = (m_1 + m_2)v
\]

Substituting the values of \( u_1 \) and \( u_2 \) from equation (2), eq.(3) becomes

\[
m_1\left(\frac{u' + v}{1 + u'v/c^2}\right) + m_2\left(\frac{-u' + v}{1 - u'v/c^2}\right) = (m_1 + m_2)v
\]
Rearranging the terms,

\[ m_1 \left( \frac{u + v}{1 + \frac{uv}{c^2}} - v \right) = m_2 \left( v - \frac{-u + v}{1 - \frac{uv}{c^2}} \right) \]

\[ m_1 \left( \frac{u + v - (uv^2/c^2)}{1 + \frac{uv}{c^2}} \right) = m_2 \left( \frac{v - \frac{-u + v}{c^2}}{1 - \frac{uv}{c^2}} \right) \]

Or,

\[ m_1 \left( \frac{u - (uv^2/c^2)}{1 + \frac{uv}{c^2}} \right) = m_2 \left( \frac{u - \frac{(uv^2)}{c^2}}{1 - \frac{uv}{c^2}} \right) \]

\[ \frac{m_1}{m_2} = \frac{1 + u v/c^2}{1 - u v/c^2} \tag{4} \]

Now, using set of equations (2), we can find RHS of equation to be equal to

\[ \frac{1 + u v/c^2}{1 - u v/c^2} = \frac{\sqrt{1 - \frac{u_2^2}{c^2}}}{\sqrt{1 - \frac{u_1^2}{c^2}}} = \frac{m_1}{m_2} \]

If \( u_2 = 0 \), i.e. \( m_2 = m_0 \), say) is at rest with respect to S frame, above equation reduces to

\[ m_1 = \frac{m_0}{\sqrt{1 - \frac{u_1^2}{c^2}}} \]

We could have chosen two masses to be identical. In that case \( m_0 \) will also be the rest mass of \( m_1 \). So we can apply above formula to a single with rest mass \( m_0 \) and moving mass \( m \), related by

\[ m = \frac{m_0}{\sqrt{1 - \frac{u^2}{c^2}}} \]

This shows that mass of a body increases with its velocity.

**Mass- Energy Equivalence**

In Newtonian Physics, mass and energy are assumed to be quite different entities. There is no mechanism in Newtonian set up, how mass and energy can be converted into each other. Like many other, it is one of unusual consequences of special relativity that mass and energy are inter-convertible into each other and hence are equivalent.

**Einstein’s Mass-energy relation \( E = mc^2 \) (Derivation)**
Consider a particle of mass $m$ acted upon by a force $F$ in the same direction as its velocity $v$. If $F$ displaces the particle through a distance $ds$, then work done $dW$ is stored as kinetic energy of the particle $dK$, therefore

$$dW = dK = F \cdot ds$$  \hspace{1cm} (1)

But from Newton’s law,

$$F = \frac{dp}{dt} = \frac{d}{dt}(mv) = m \frac{dv}{dt} + v \frac{dm}{dt}$$ \hspace{1cm} (2)

Using (2) in (1)

$$dK = m \frac{dv}{dt} ds + v \frac{dm}{dt} ds = m \frac{ds}{dt} dv + v \frac{ds}{dt} dm$$

Or,

$$dK = mv dv + v^2 dm$$ \hspace{1cm} (3)

Now we have,

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} = m_0(1 - \frac{v^2}{c^2})^{-1/2}$$ \hspace{1cm} (4)

Taking the differential,

$$dm = m_0(-1/2)(1 - \frac{v^2}{c^2})^{-3/2} \left(-\frac{2v}{c^2} \right) dv = \frac{m_0 v dv}{c^2(1 - \frac{v^2}{c^2})^{3/2}}$$

But

$$m_0 = m(1 - \frac{v^2}{c^2})^{1/2}$$

$$dm = \frac{m(1 - \frac{v^2}{c^2})^{1/2} v dv}{c^2(1 - \frac{v^2}{c^2})^{3/2}}$$

Or

$$dm = \frac{mv dv}{(c^2 - v^2)}$$

or,

$$mv dv = (c^2 - v^2) dm$$ \hspace{1cm} (5)

Using eq (5) in eq (3)

$$dK = (c^2 - v^2)dm + v^2 dm = c^2 dm$$

Let the change in kinetic energy of the particle be $K$, as its mass changes from rest mass $m_0$ to effective mass $m$, then

$$K = \int_0^K dK = \int_{m_0}^m c^2 dm = c^2(m - m_0) = c^2 \left( \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}} - m_0 \right)$$

This is the relativistic expression for kinetic energy of a particle. It says that kinetic energy of a particle is due to the increase in mass of the particle on account of its relative motion and is equal to the product of
the gain in mass and square of the velocity of light, \( m_0 c^2 \) can be regarded as the rest energy of the particle of rest mass \( m_0 \). The total energy \( E \) of a moving particle is the sum of kinetic energy and its rest mass energy.

\[
E = m_0 c^2 + (m - m_0)c^2 = mc^2
\]

Or,

\[
E = mc^2
\]

This is the celebrated mass-energy equivalence relation. This equation is so famous that even common men identify Einstein with it.

**Zero Rest Mass Particles/Mass-less Particle: Photon**

We can not imagine a zero mass particle in classical physics. But SR allows particles with zero rest mass such as photon.

We have the energy-momentum relation,

\[
E = \sqrt{m_0^2 c^4 + (p^2 c^2)}
\]

(1)

For a massless particle, \( m_0 = 0 \)

Therefore,

\[
E = pc \quad \text{or} \quad p = E/c
\]

(2)

But \( p = mv \), therefore

\[
\frac{E}{c^2} = \frac{E}{c}
\]

Which implies \( v = c \)

This says that a massless particles always move with the speed of light. Energy and momentum of a massless particle is given by equation (2). Massless particles can exist only as long as the move at the speed of light. Examples are photon, neutrinos and theoretically predicted gravitons.

**Significance**: This equation represents that energy can neither be created nor be destroyed, but it can change its form.

**Example**: Pair Annihilation: In pair annihilation, electron and positron reacts to release photons.

\[
e^- + e^+ \rightarrow \gamma
\]

As electron and positron have mass but photon has energy but not mass. Therefore, here mass is changed into energy. The opposite of this reaction is called pair production.
**Continuity equation for current density**

Statement: Equation of continuity represents the law of conservation of charge. That is the charge flowing out (i.e. current) through a closed surface in some volume is equal to the rate of decrease of charge within the volume:

\[ I = -\frac{dq}{dt} \]  \hspace{1cm} (1)

where \( I \) is current flowing out through a closed surface in a volume and \(-\frac{dq}{dt}\) is the rate of decrease of charge within the volume.

As \( I = \int \int J \cdot ds \) and \( q = \int \int \int \rho \cdot dv \) where \( J \) is the Conduction current density and \( \rho \) is the Volume charge density.

Substituting the value of \( I \) and \( q \) in equation (1), it will become

\[ \int \int J \cdot ds = -\int \int \int \frac{d\rho}{dt} \cdot dv \]  \hspace{1cm} (2)

Apply Gauss’s Divergence Theorem to L.H.S. of above equation to change surface integral to volume integral,

\[ \int \int \int \text{divergence (} J \text{)} \cdot dV = -\int \int \int (\frac{d\rho}{dt}) \cdot dv \]

As two volume integrals are equal only if their integrands are equal

\( \text{divergence (} J \text{)} = -\frac{d\rho}{dt} \)

This is equation of continuity for time varying fields.

**Equation of Continuity for Steady Currents:** As \( \rho \) does not vary with time for steady current that is \( \frac{d\rho}{dt} = 0 \)

\( \text{divergence (} J \text{)} = 0 \)

The above equation is the equation of continuity for steady currents.
**Differential form of Maxwell’s equations**

**First equation**

It states that the total electric flux $\phi_E$ passing through a closed hypothetical surface is equal to $1/\varepsilon_0$ times the net charge enclosed by the surface i.e, $\oint E \cdot dS = q/\varepsilon_0$ or $\oint D \cdot dS = \iiint \rho dV$ (1)

Apply Gauss’s Divergence theorem to change L.H.S. of equation (1) from surface integral to volume integral $\oint D \cdot dS = \iiint (\nabla \cdot D) dV$

Substituting this equation in equation (1), we get $\iiint (\nabla \cdot D) dV = \iiint \rho dV$

As two volume integrals are equal only if their integrands are equal. Thus, $\nabla \cdot D = \rho$ (2)

Equation (2) is the **Differential form of Maxwell’s first equation**.

**Second equation**

It states that the total magnetic flux $\phi_m$ emerging through a closed surface is always equal to zero. $\oint B \cdot dS = 0$ (3)

Apply Gauss’s Divergence theorem $\oint B \cdot dS = \iiint (\nabla \cdot B) dV$

Putting this in equation (3) $\iiint (\nabla \cdot B) dV = 0$

Thus, $\nabla \cdot B = 0$ (4)

The equation (4) is **differential form of Maxwell’s second equation**.

**Third Equation**

a) It states that, whenever magnetic flux linked with a circuit changes then induced electromotive force (emf) is set up in the circuit. This induced emf lasts so long as the change in magnetic flux continues.

(b) The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the circuit.
Therefore, induced emf= \(-\frac{d\phi_m}{dt}\) \hspace{1cm} (5)

Where \(\phi_m=\oiint B.dS\) \hspace{1cm} (6)

Here negative sign is because of Lenz’s law which states that the induced emf set up a current in such a direction that the magnetic effect produced by it opposes the cause producing it.

Also, definition of emf states that emf is the closed line integral of the non-conservative electric field generated by the battery.

That is \(\text{emf}=\oint E.dl\) \hspace{1cm} (7)

Putting equations (5) and (6), in equation (4) we get

\(\oint E.dl=\int \frac{dB}{dt}.dS\) \hspace{1cm} (8)

Apply Stoke’s theorem to L.H.S. of equations (8) to change line integral to surface integral. That is \(\oint E.dl=\oiint (\nabla \times E).dS\)

By substituting above equation in equation(8), we get

\(\oiint (\nabla \times E).dS=-\oiint \frac{dB}{dt}.dS\)

As two surface integral are equal only when their integrands are equal.

Thus \(\nabla \times E=-\frac{dB}{dt}\) \hspace{1cm} (9) \hspace{1cm} This is the differential form of Maxwell’s 3rd equation.

**Forth Equation (Displacement current, Modifying equation for the curl of magnetic field to satisfy continuity equation) Modified Ampere’s Circuital Law**

Here the first question arises, why there was need to modify Ampere’s circuital Law?

To give answer to this question, let us first discuss Ampere’s law (without modification)

**Statement of Ampere’s circuital law (without modification):** It states that the line integral of the magnetic field \(H\) around any closed path or circuit is equal to the current enclosed by the path.

That is \(\oint H.dl=I\)

Let the current is distributed through the surface with a current density \(J\)
Then \[ I = \int \int J \cdot dS \]

This implies that \[ \oint H \cdot dl = \int \int J \cdot dS \] (10)

Applying Stoke’s theorem to L.H.S. of equation (10) to change line integral to surface integral \[ \oint H \cdot dl = \iint (\nabla \times H) \cdot dS \]

Substituting above equation in equation(10), we get \[ \iint (\nabla \times H) \cdot dS = \int \int J \cdot dS \]

As two surface integrals are equal only if their integrands are equal

Thus , \[ \nabla \times H = J \] (11)

This is the differential form of Ampere’s circuital Law (without modification) for steady currents.

Taking divergence of equation (10) \[ \nabla \cdot (\nabla \times H) = \nabla \cdot J \]

As divergence of the curl of a vector is always zero, therefore \[ \nabla \cdot (\nabla \times H) = 0 \]

It means \[ \nabla \cdot J = 0 \]

Now, this is equation of continuity for steady current but not for time varying fields, as equation of continuity for time varying fields is \[ \nabla \cdot J = \frac{d \rho}{dt} \]

So, there is inconsistency in Ampere’s circuital law. This is the reason that led Maxwell to modify: Ampere’s circuital law.

Modification of Ampere’s circuital law: Maxwell modified Ampere’s law by giving the concept of displacement current \( D \) and so the concept of displacement current density \( J_d \) for time varying fields.

He concluded that equation (10) for time varying fields should be written as \[ \nabla \times H = J + J_d \] (12)

By taking divergence of equation(12), we get

\[ \nabla \cdot (\nabla \times H) = \nabla \cdot J + \nabla \cdot J_d \]

As divergence of the curl of a vector is always zero, therefore
\(\nabla \cdot (\nabla \times \mathbf{H}) = 0\)

It means, \(\nabla \cdot (\mathbf{J} + \mathbf{J}_d) = 0\) Or \(\nabla \cdot \mathbf{J} = -\nabla \cdot \mathbf{J}_d\)

But from equation of continuity for time varying fields, \(\nabla \cdot \mathbf{J} = -\frac{d\mathbf{p}}{dt}\)

By comparing above two equations of \(\mathbf{j}\), we get

\[
\nabla \cdot \mathbf{j}_d = \frac{d(\nabla \cdot \mathbf{D})}{dt}
\]

(13)

Because from maxwells first equation \(\nabla \cdot \mathbf{D} = \mathbf{p}\)

As the divergence of two vectors is equal only if the vectors are equal.

Thus \(\mathbf{J}_d = \frac{d\mathbf{D}}{dt}\)

Substituting above equation in equation (12), we get

\[
\nabla \times \mathbf{H} = \mathbf{J} + \frac{d\mathbf{D}}{dt}
\]

(14)

Here, \(\mathbf{J}_d = \frac{d\mathbf{D}}{dt}\) =Displacement current density
\(\mathbf{J}\)=conduction current density, \(\mathbf{D}\) = displacement current

The equation(14) is the **Differential form of Maxwell’s fourth equation** or Modified Ampere’s circuital law.

**Integral form of Maxwell’s equations**

**First Equation:** It states that the total electric flux \(\Phi_E\) passing through a closed hypothetical surface is equal to \(1/\varepsilon_0\)times the net charge enclosed by the surface:

\[
\Phi_E = \oint \mathbf{E} \cdot d\mathbf{S} = \frac{q}{\varepsilon_0}
\]

(1)

\[
\oint \mathbf{D} \cdot d\mathbf{S} = q
\]

(2)

Where \(\mathbf{D} = \varepsilon_0 \mathbf{E}\) = Displacement vector
Let the charge be distributed over a volume \(V\) and \(p\) be the volume charge density. Therefore, \(q = \int p dV\)

Therefore \(\oint \mathbf{D} \cdot d\mathbf{S} = \int p dV\)
Equation (2) is the integral form of Maxwell’s first equation or Gauss’s law in electrostatics.

**Second equation:** It states that the total magnetic flux $\varphi_m$ emerging through a closed surface is zero. i.e, $\varphi_m = \oint B \cdot dS = 0$ \hspace{1cm} (3)

The equation (3) is the integral form of Maxwell’s second equation. This equation also proves that magnetic monopole does not exist.

**Third Equation:** (a) It states that whenever magnetic flux linked with a circuit changes then induced electromotive force (emf) is set up in the circuit. This induced emf lasts so long as the change in magnetic flux continues.

(b) The magnitude of induced emf is equal to the rate of change of magnetic flux linked with the circuit.

Therefore, induced emf = $-\frac{d\varphi_m}{dt}$ \hspace{1cm} (4)

Where $\varphi_m = \int B \cdot dS$ \hspace{1cm} (5)

Here negative sign is because of Lenz’s law which states that the induced emf set up a current in such a direction that the magnetic effect produced by it opposes the cause producing it.

**Also definition of emf states** that emf is the closed line integral of the non-conservative electric field generated by the battery.

That is $\text{emf} = \oint E \cdot dl$ \hspace{1cm} (6)

Putting equations (5) and (6), in equation (4) we get

$\oint E \cdot dl = -\int \frac{dB}{dt} \cdot t \cdot dS$ \hspace{1cm} (7)

Equation (7) is the integral form of Maxwell’s third Equation or Faraday’s law of electromagnetic induction.

**Fourth equation/modified Ampere’s circuital Law:** The line integral of the
Magnetic field $H$ around any closed path or circuit is equal to the conduction current plus the time derivative of electric displacement through any surface bounded by the path i.e.,

$$\oint H \cdot dl = \int (J + \frac{dD}{dt}) \cdot dS$$  

(8)

Equation (8) is the integral form of Maxwell’s fourth equation.

**Concept of Displacement Current (Difference Between Displacement Current and Conduction Current)**

Let there be a parallel R-C network with a voltage $V$ as shown in fig. Let the current through resistor $R$ is $I_c$ and by Ohm’s law it is given by

$$I_c = \frac{V}{R}$$

And current through capacitor $C$ is $I_d$ and is given by

$$I_d = \frac{dQ}{dt}$$

$$I_d = \frac{dQ}{dt} = \frac{C}{d} \frac{dv}{dt}$$  

(1)

In practice, the current does not flow through the capacitor. But, the current that flows out of one electrode of capacitor equals the current that flows in to the other electrode. The net effect is as if there is a current flowing through the path containing the capacitor. But current, $I_c$ actually flows through the resistor.

Hence, from the above result, current flowing through the resistor is known as conduction current and it obeys Ohm’s law, while the current flowing through the capacitor is commonly known as Displacement current.

**Mathematical Proof:** As the electric field inside each element equals the voltage $V$ across the element divided by its length $d$

That is

$$E = \frac{V}{d} \text{ or } V = Ed$$  

(2)

Now the current density in resistor is given by

$$J_c = I_c / A = \sigma E$$  

(3)

Where $A$ = cross-sectional area, $\sigma$ = conductivity of resistance element

Also capacitance of a parallel plate capacitor is given by

$$C = \varepsilon_0 A / d$$  

(4)

Now rewrite equation (1)

$$I_d = C \frac{dV}{dt}$$
By substituting the values of $V$ and $C$ from equations (2 and 4) in above equation, we get

$$I_d = \varepsilon_0 A/d \ (E/t)$$  
$$I_d = \varepsilon_0 A \ E/t$$  \hspace{1cm} (5)

Therefore current density $J_d$ inside capacitor is

$$J_d = I_d/A$$

Substituting value of $I_d$ from equation (5) in above equation, we get

$$J_d = \varepsilon_0 A \ E/t$$

Or $J_d = \varepsilon_0 A \ E/t$ \hspace{1cm} (6a)

Or $J_d = D/t$ \hspace{1cm} (6b)

Where $D = \varepsilon_0 E$ = Electric displacement vector and $J_d$ = Displacement current density

Equation (6a) proves that displacement current density arises whenever there will be change in electric field $E$ that is $(E/t \neq 0)$.

**Wave Equation in Free Space**

Our light wave is basically electromagnetic wave (EM) wave, so as the name suggests it consists of electric and magnetic components. We have two electric field vectors $E$, $D$ and two magnetic $H$, $B$. Maxwell eqs. are basically for the EM waves. As these vectors can curl and diverge, so Maxwell eqs. consist of these 4 equations covering 4 vectors and their behaviour. So these equations, covered, Gauss law, faradays law and modified amperes circuital law.

Free space or non-conducting or lossless or in general perfect dielectric medium has following characteristics:

(a) No condition current i.e $\sigma=0$, thus $J=0$ ($J=\sigma E$)

(b) No charges (i.e $\rho=0$)

**Wave Equation in Terms of Electric Field Intensity, $E$**

Therefore for the above cases, Maxwell’s equations will become

$$\nabla \cdot D=0 \text{ or } \nabla \cdot E=0$$ \hspace{1cm} ($\rho=0$) \hspace{1cm} (1a)

$$\nabla \cdot B=0 \text{ or } \nabla \cdot H=0$$ \hspace{1cm} (1b)

$$\nabla \times E= -dB/dt \text{ or } \nabla \times E= -\mu dH/dt$$ \hspace{1cm} (1c)
Now taking curl of third Maxwell’s equation (1c), we get

\[ \nabla \times (\nabla \times E) = -\mu \varepsilon (\nabla \times H)/dt \]

Applying standard vector identity, that is \[ \nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - \nabla^2 E \] on left hand side of above equation, we get

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -\mu \varepsilon \nabla \times H)/dt \]

Substituting equations (1a) and (1d) in equations (2) ,we get

\[ -\nabla^2 E = -\mu \varepsilon \partial^2 E/\partial t^2 \]

Or \[ \nabla^2 E = \mu_0 \varepsilon_0 \partial^2 E/\partial t^2 \]

Equation (5) is the required wave equation in terms of electric field intensity, \( E \) for free space. This is the equation that \( E \) must obey.

If \( \mu = \mu_0 \) and \( \varepsilon = \varepsilon_0 \), equations (3) and (5) will become

\[ \nabla^2 E = \mu_0 \varepsilon_0 \partial^2 E/\partial t^2 \]

in free space

**Wave Equation In terms of Magnetic Field Intensity, \( H \)**

Take curl of fourth Maxwell’s equation (1d), we get

\[ \nabla \times (\nabla \times H) = \varepsilon \partial (\nabla \times E)/\partial t \]

Applying standard vector identity that is \[ \nabla \times (\nabla \times H) = \nabla (\nabla \cdot H) - \nabla^2 H \]

On left side of above equation, we get

\[ \nabla (\nabla \cdot H) - \nabla^2 H = \varepsilon \partial (\nabla \times E)/\partial t \]

Substituting equations (1b) and (1c) in equation (4), we get
\[-\nabla^2 H = - \mu \varepsilon \frac{d^2 H}{dt^2}\]

Or \[\nabla^2 H = \mu \varepsilon \frac{d^2 H}{dt^2}\]  

Equations (5) is the required wave equation in terms of magnetic field intensity, H and this is the law that H must obey.

In free space \(\mu = \mu_0\) and \(\varepsilon = \varepsilon_0\), equations (3) and (5) will become \(\nabla^2 H = \mu_0 \varepsilon_0 \frac{d^2 H}{dt^2}\).

**Important features or characteristics or facts about electromagnetic waves:**

(i) Electromagnetic waves are produced by accelerated or oscillating charges.

(ii) They don’t require any material medium for their propagation.

(iii) They travel in free space with a speed of light.

(iv) A sinusoidal variation occurs in both the electric and magnetic field vectors.

(v) Electromagnetic waves are transverse in nature.

(vi) Velocity of em waves depends on electric and magnetic properties of medium through which travel and independent of amplitude of field vectors.

(vii) Velocity of em waves in a dielectric is less than velocity of light.

(viii) Electromagnetic waves carry energy which is distributed equally between electric and magnetic fields.

(ix) The electric vector is responsible for the optical effects of an em waves so it is known as the light vector.

(x) Electromagnetic waves are uncharged so they are not deflected by electric and magnetic fields.

**Wave Equation in Non-conducting Medium**
Wave Equation in Terms of Electric Field Intensity, $E$
Non-conducting or in general perfect dielectric medium has following characteristics:

(a) No condition current i.e $\sigma=0$, thus $J=0$ ($J=\sigma E$)

(b) No charges i.e $\rho=0$

Therefore for the above cases, Maxwell’s equations will become

\[ \nabla \cdot D=0 \quad \text{or} \quad \nabla \cdot E=0 \quad (\rho=0) \quad (1a) \]

\[ \nabla \cdot B=0 \quad \text{or} \quad \nabla \cdot H=0 \quad (1b) \]

\[ \nabla \times E = -\frac{d}{dt} \mathbf{B} \quad \text{or} \quad \nabla \times E = -\mu \frac{d}{dt} \mathbf{H} \quad (1c) \]

\[ \nabla \times H = \frac{d}{dt} \mathbf{D} \quad \text{or} \quad \nabla \times H = \epsilon \frac{d}{dt} \mathbf{E} \quad (J=0) \quad (1d) \]

Now taking curl of third Maxwell’s equation (1c), we get

\[ \nabla \times (\nabla \times E) = -\mu \frac{d}{dt} (\nabla \times H) \]

Applying standard vector identity, that is \[ \nabla \times (\nabla \times E) = \nabla (\nabla \cdot E) - \nabla^2 E \] on left hand side of above equation, we get

\[ \nabla (\nabla \cdot E) - \nabla^2 E = -\mu \frac{d}{dt} (\nabla \times H) \quad (2) \]

Substituting equations (1a) and (1d) in equations (2), we get

\[-\nabla^2 E = -\mu \epsilon \frac{d^2}{dt^2} \mathbf{E} \]

Or

\[ \nabla^2 E = \mu \epsilon \frac{d^2}{dt^2} \mathbf{E} \quad (3) \]

Equation (5) is the required wave equation in non-conducting medium in terms of electric field intensity, $E$ for free space. This is the equation that $E$ must obey.

Wave Equation In terms of Magnetic Field Intensity, $H$
Take curl of fourth Maxwell’s equation (1d), we get

\[ \nabla \times (\nabla \times H) = \epsilon \frac{d}{dt}(\nabla \times E) \]
Applying standard vector identity that is \[\nabla \times (\nabla \times \mathbf{H}) = \nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H}\]

On left side of above equation, we get

\[\nabla (\nabla \cdot \mathbf{H}) - \nabla^2 \mathbf{H} = \varepsilon \frac{d(\nabla \times \mathbf{E})}{dt}\]  
(4)

Substituting equations (1b) and (1c) in equation (4), we get

\[-\nabla^2 \mathbf{H} = -\mu \varepsilon \frac{d^2 \mathbf{H}}{dt^2}\]

Or \[\nabla^2 \mathbf{H} = \mu \varepsilon \frac{d^2 \mathbf{H}}{dt^2}\]  
(5)

Equations (5) is the required wave equation in non-conducting in terms of magnetic field intensity, \(\mathbf{H}\) and this is the law that \(\mathbf{H}\) must obey.

**Wave Equation in conducting Medium**

Consider the propagation of an electromagnetic wave through a conducting medium which obeys Ohm's law: \(\mathbf{j} = \sigma \mathbf{E}\).

Here, \(\sigma\) is the **conductivity** of the medium in question. Maxwell's equations for the wave take the form:

\[\nabla \cdot \mathbf{E} = 0,\]

\[\nabla \cdot \mathbf{B} = 0,\]

\[\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t},\]

\[\nabla \times \mathbf{B} = \mu_0 \mathbf{j} + \varepsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t},\]

where \(\varepsilon\) is the dielectric constant of the medium. It follows, from the above equations, that
\[ \nabla \times \nabla \times \mathbf{E} = -\nabla^2 \mathbf{E} = -\frac{\partial \nabla \times \mathbf{B}}{\partial t} = -\frac{\partial}{\partial t} \left[ \mu_0 \sigma \mathbf{E} + \epsilon \epsilon_0 \mu_0 \frac{\partial \mathbf{E}}{\partial t} \right]. \]

Looking for a wave-like solution of the form

\[ \mathbf{E} = E_0 e^{i(kz-\omega t)}, \]

we obtain the dispersion relation

\[ k^2 = \mu_0 \omega (\epsilon \epsilon_0 \omega + i \sigma). \]

Consider a “poor” conductor for which \( \approx \epsilon \epsilon_0 \omega \). In this limit, the dispersion relation yields

\[ k \approx n \frac{\omega}{c} + i \frac{\sigma}{2} \sqrt{\frac{\mu_0}{\epsilon \epsilon_0}}, \]

where \( n = \sqrt{\epsilon} \) is the refractive index. Substitution in the wave solution

\[ \mathbf{E} = E_0 e^{-z/d} e^{i(k_r z-\omega t)}, \]

Where

\[ d = \frac{2}{\sigma} \sqrt{\frac{\epsilon \epsilon_0}{\mu_0}}, \]

and

\[ k_r = n \omega / c. \]

Thus, we conclude that the amplitude of an electromagnetic wave propagating through a conductor decays exponentially on some length-scale, \( d \), which is termed the skin-depth. Note, from the above Equation that the skin-depth for a poor conductor is independent of the frequency of the wave. Note, also, that for a poor conductor, indicating that the wave penetrates many wave-lengths into the conductor before decaying away.

Consider a “good” conductor for which \( \sigma \gg \epsilon \epsilon_0 \omega \). In this limit, the dispersion relation yields

\[ k \approx \sqrt{i \mu_0 \sigma \omega}. \]

Substitution this skin depth becomes

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It can be seen that the skin-depth for a good conductor decreases with increasing wave frequency. The fact that indicates that the wave only penetrates a few wave-lengths into the conductor before decaying away.

**Skin Depth**

Skin depth or depth of penetration is defined as the depth in which the strength of electric field associated with electromagnetic wave reduce to 1/e times of its initial value. The figure below shows the variation of $E$ with distance of em wave inside a conducting medium and the value of skin depth inside a good conductor is given by

$$d = \frac{1}{k_r} = \sqrt{\frac{2}{\mu_0 \sigma \omega}}.$$

**Poynting Theorem**

**Statement**: This theorem states that the cross product of electric field vector, $E$ and magnetic field vector, $H$ at any point is a measure of the rate of flow of electromagnetic energy per unit area at that point, that is
\[ P = E \times H \]

Here \( P \) → Poynting vector and it is named after its discoverer, J.H. Poynting. The direction of \( P \) is perpendicular to \( E \) and \( H \) and in the direction of vector \( E \times H \)

**Proof:** Consider Maxwell’s fourth equation (Modified Ampere’s Circuital Law), that is

\[ \nabla \times H = J + \varepsilon \frac{dE}{dt} \]

or

\[ J = (\nabla \times H) - \varepsilon \frac{dE}{dt} \]

The above equation has the dimensions of current density. Now, to convert the dimensions into rate of energy flow per unit volume, take dot product of both sides of above equation by \( E \), that is

\[ E \cdot J = E \cdot (\nabla \times H) - \varepsilon E \cdot \frac{dE}{dt} \]  \hspace{1cm} (1)

Use vector Indentity

\[ \nabla \cdot (E \times H) = H \cdot (\nabla \times E) - E \cdot (\nabla \times H) \]

or

\[ E \cdot (\nabla \times H) = H \cdot (\nabla \times E) - \nabla \cdot (E \times H) \]

By substituting value of \( E \cdot (\nabla \times H) \) in equation (1), we get

\[ E \cdot J = H \cdot (\nabla \times E) - \nabla \cdot (E \times H) - \varepsilon E \cdot \frac{dE}{dt} \]  \hspace{1cm} (2)

Also from Maxwell’s third equation (Faraday’s law of electromagnetic induction).

\[ \nabla \times E = \mu \frac{dH}{dt} \]

By substituting value of \( \nabla \times E \) in equation (2) we get

\[ E \cdot J = \mu H \frac{dH}{dt} - \varepsilon E \cdot \frac{dE}{dt} - \nabla \cdot (E \times H) \]  \hspace{1cm} (3)

We can write

\[ H \cdot \frac{dH}{dt} = 1/2 \frac{dH^2}{dt} \]  \hspace{1cm} (4a)

\[ E \cdot \frac{dE}{dt} = 1/2 \frac{dE^2}{dt} \]  \hspace{1cm} (4b)
By substituting equations 4a and 4b in equation 3, we get

\[ \mathbf{E} \cdot \mathbf{J} = -\mu/2 \frac{d\mathbf{H}^2}{dt} - \varepsilon/2 \frac{d\mathbf{E}^2}{dt} - \nabla \cdot (\mathbf{E} \times \mathbf{H}) \]

\[ \mathbf{E} \cdot \mathbf{J} = -d(\mu\mathbf{H}^2/2 + \varepsilon\mathbf{E}^2/2)/dt - \nabla \cdot (\mathbf{E} \times \mathbf{H}) \]

By taking volume integral on both sides, we get

\[ \iiint \mathbf{E} \cdot \mathbf{J} \, dV = \frac{d[\iiint (\mu\mathbf{H}^2/2 + \varepsilon\mathbf{E}^2/2) \, dV]}{dt} - \iiint \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, dV \]  \hspace{1cm} (5)

apply Gauss’s Divergence theorem to second term of R.H.S., to change volume integral into surface integral, that is

\[ \iiint \nabla \cdot (\mathbf{E} \times \mathbf{H}) \, dV = \oiint (\mathbf{E} \times \mathbf{H}) \cdot dS \]

Substitute above equation in equation (5)

\[ \iiint \mathbf{E} \cdot \mathbf{J} \, dV = \frac{d[\iiint (\mu\mathbf{H}^2/2 + \varepsilon\mathbf{E}^2/2) \, dV]}{dt} - \oiint (\mathbf{E} \times \mathbf{H}) \cdot dS \] \hspace{1cm} (6)

or \[ \oiint (\mathbf{E} \times \mathbf{H}) \cdot dS = \frac{d[\iiint (\mu\mathbf{H}^2/2 + \varepsilon\mathbf{E}^2/2) \, dV]}{dt} - \iiint \mathbf{E} \cdot \mathbf{J} \, dV \]

Interpretation of above equation:

L.H.S. Term

\[ \oiint (\mathbf{E} \times \mathbf{H}) \cdot dS \rightarrow \text{It represents the rate of outward flow of energy through the surface of a volume V and the integral is over the closed surface surrounding the volume. This rate of outward flow of power from a volume V is represented by} \]

\[ \oiint \mathbf{P} \cdot dS = \oiint (\mathbf{E} \times \mathbf{H}) \cdot dS \]

where Poynting vector, \( \mathbf{P} = \mathbf{E} \times \mathbf{H} \)
Inward flow of power is represented by \(- \iint \mathbf{P} \cdot dS = - \iint \mathbf{(E} \times \mathbf{H}) \cdot ds\)

**R.H.S. First Term**

\(- \iiint \mathbf{d} \left( \frac{\mu \mathbf{H}^2}{2} + \varepsilon \mathbf{E}^2 / 2 \right) / dt \mathbf{d}V \rightarrow \) If the energy is flowing out of the region, there must be a corresponding decrease of electromagnetic energy. So here negative sign indicates decrease. Electromagnetic energy is the sum of magnetic energy, \(\mu \mathbf{H}^2 / 2\) and electric energy, \(\varepsilon \mathbf{E}^2 / 2\). So, first term of R.H.S. represents rate of decrease of stored electromagnetic energy.

**R.H.S. Second Term**

\(- \iiint \mathbf{E} \cdot \mathbf{J} \mathbf{d}V \rightarrow \) Total ohmic power dissipated within the volume.

So, from the law of conservation of energy, equation (6) can be written in words as

rate of energy dissipation in volume \(V = \) Rate at which stored electromagnetic energy is decreasing in \(V\) + Inward rate of flow of energy through the surface of the volume.
**BLACKBODY RADIATION SPECTRUM**

A blackbody is an object that absorbs all of the radiation that it receives (that is, it does not reflect any light, nor does it allow any light to pass through it and out the other side). The energy that the blackbody absorbs heats it up, and then it will emit its own radiation. The only parameter that determines how much light the blackbody gives off, and at what wavelengths, is its temperature. There is no object that is an ideal blackbody, but many objects (stars included) behave approximately like blackbodies. Other common examples are the filament in an incandescent light bulb or the burner element on an electric stove. As you increase the setting on the stove from low to high, you can observe it produce blackbody radiation; the element will go from nearly black to glowing red hot.

![Plot of the spectrum of a blackbody with different temperatures](image)

*The spectrum of a blackbody is continuous (it gives off some light at all wavelengths), and it has a peak at a specific wavelength. The peak of the blackbody curve in a spectrum moves to shorter wavelengths for hotter objects.*

**Rayleigh-Jeans radiation law**

The Rayleigh-Jeans Radiation Law was a useful but not completely successful attempt at establishing the functional form of the spectra of thermal radiation. The energy density $u_\nu$ per unit frequency interval at a frequency $\nu$ is, according to the Rayleigh-Jeans Radiation,

$$u_\nu = \frac{8\pi \nu^2 kT}{c^3} \, d\nu$$

where $k$ is Boltzmann's constant, $T$ is the absolute temperature of the radiating body and $c$ is the speed of light in a vacuum. This formula fits the empirical measurements for low frequencies but fails increasingly for higher frequencies. The failure of the formula to match the new data was called the ultraviolet catastrophe. The significance of this inadequate so-called law is that it provides an asymptotic condition which other proposed formulas, such as Planck's, need to satisfy. It gives a value to an otherwise arbitrary constant in Planck's thermal radiation formula.

**Wien’s Displacement Law**

When the temperature of a blackbody radiator increases, the overall radiated energy increases and the peak of the radiation curve moves to shorter wavelengths. When the maximum is evaluated from the Planck radiation formula, the product of the peak wavelength and the temperature is found to be a constant.

$$\lambda_m T = 2.898 \times 10^{-3} \text{ mK}$$
Assumption of Quantum Theory of Radiation

The distribution of energy in the spectrum of radiations of a hot body cannot be explained by applying the classical concepts of physics. Max Planck gave an explanation to this observation by his Quantum Theory of Radiation. His theory says:

a) The radiant energy is always in the form of tiny bundles of light called quanta i.e. the energy is absorbed or emitted discontinuously.
b) Each quantum has some definite energy $E=\hbar\nu$, which depends upon the frequency ($\nu$) of the radiations. where, $\hbar =$ Planck’s constant $= 6.626 \times 10^{-34}$ Js.
c) The energy emitted or absorbed by a body is always a whole multiple of a quantum i.e. $n\hbar$This concept is known as quantization of energy.

Planck’s Law (or Planck’s Radiation Law)

Planck's radiation law is derived by assuming that each radiation mode can be described by a quantized harmonic oscillator with energy $E_n = n\hbar\nu$

Let $N_0$ be the number of oscillators with zero energy i.e $E_0$ (in the so-called ground-state), then the numbers in the 1st, 2nd, 3rd etc. levels ($N_1$, $N_2$, $N_3$...) are given by:

$$N_1 = N_0 \exp\left(\frac{-E_1}{kT}\right) ; N_2 = N_0 \exp\left(\frac{-E_2}{kT}\right) ; N_3 = N_0 \exp\left(\frac{-E_3}{kT}\right)$$

But since $E_n = n\hbar\nu$
The total number of oscillators \( N = N_0 + N_1 + N_2 + N_3 \ldots \)

The total energy \( E = h \nu N_0 \exp \left( \frac{-h \nu}{kT} \right) + 2h \nu N_0 \exp \left( \frac{-2h \nu}{kT} \right) + 3h \nu N_0 \exp \left( \frac{-3h \nu}{kT} \right) + \ldots \)

The avg. energy \( \langle E \rangle = \frac{E}{N} \)

\[
\langle E \rangle = \frac{h \nu N_0 \left[ \exp \left( \frac{-h \nu}{kT} \right) + 2\exp \left( \frac{-2h \nu}{kT} \right) + 3\exp \left( \frac{-3h \nu}{kT} \right) + \ldots \right] }{N_0 \left[ 1 + \exp \left( \frac{-h \nu}{kT} \right) + \exp \left( \frac{-2h \nu}{kT} \right) + \ldots \right]} 
\]

\[
\langle E \rangle = \frac{h \nu \exp \left( \frac{-h \nu}{kT} \right)}{\left[ 1 - \exp \left( \frac{-h \nu}{kT} \right) \right]}
\]

\[
\langle E \rangle = \frac{h \nu}{\exp \left( \frac{h \nu}{kT} \right) - 1}
\]

According to Rayleigh-Jeans law, using classical physics, the energy density \( u_\nu \) per frequency interval was given by:

\[
u_\nu = \frac{8\pi \nu^2 kT}{c^3}d\nu
\]

where \( kT \) was the energy of each mode of the electromagnetic radiation. We need to replace the \( kT \) in this equation with the average energy for the harmonic oscillators that we have just derived above. So, we re-write the energy density as

\[
\nu_\nu = \frac{8\pi \nu^2}{c^3} \frac{h \nu}{\exp \left( \frac{h \nu}{kT} \right) - 1}d\nu
\]

**Comparison of Rayleigh Jeans, Wiens & Planck’s Radiation Laws**

**WAVE PARTICLE DUALITY**

In physics and chemistry, wave-particle duality holds that light and matter exhibit properties of both waves and of particles. A central concept of quantum mechanics,
duality addresses the inadequacy of conventional concepts like "particle" and "wave" to meaningfully describe the behaviour of quantum objects.

Publicized early in the debate about whether light was composed of particles or waves, a wave-particle dual nature soon was found to be characteristic of electrons as well. The evidence for the description of light as waves was well established at the turn of the century when the *photoelectric effect* introduced firm evidence of a *particle nature of light* as well. On the other hand, the particle properties of electron was well documented when the DeBroglie hypothesis and the subsequent experiments by Davisson and Germer established the *wave nature of the electron*.

**De Broglie Matter Waves**

**De Broglie Hypothesis**

In 1924, Lewis de-Broglie proposed that matter has dual characteristic just like radiation. His concept about the dual nature of matter was based on the following observations:

(a) The whole universe is composed of matter and electromagnetic radiations. Since both are forms of energy so can be transformed into each other.

(b) The matter loves symmetry. As the radiation has dual nature, matter should also possess dual character.

According to the de Broglie concept of matter waves, the matter has dual nature. It means when the matter is moving it shows the wave properties (like interference, diffraction etc.) are associated with it and when it is in the state of rest then it shows particle properties. Thus the matter has dual nature. The waves associated with moving particles are matter waves or de-Broglie waves.

\[ \lambda = \frac{h}{mv} \]

where, \( \lambda \) = de Broglie wavelength associated with the particle, \( h \) = Planck’s constant, \( m \) = relativistic mass of the particle and \( v \) = velocity of the particle

**Phase velocity/Group Velocity**

The **phase velocity** of a wave is the rate at which the phase of the wave propagates in space. This is the velocity at which the phase of any one frequency component of the wave travels.
where,

\[ v = \text{velocity of the particle associated with the wave.} \]

For Photon \( v = c \) in vacuum \( \rightarrow V_p = c \)

For other particles \( v < c \) \( \rightarrow V_p > c \) which has no physical significance.

For a physically significant representation of matter waves, we require a wave packet to be associated with the moving particle (or body).

Consider two waves of same amplitude \( A \) but slightly different wave numbers and angular frequencies represented by the equations,

\[ y_1 = A \sin(kx - \omega t) \quad \text{and} \quad y_2 = A \sin[(k + \Delta k)x - (\omega + \Delta \omega)t] \]

The resultant of the superposition of these two waves can be written as,

\[ y = y_1 + y_2 = 2A \cos \left( \frac{\Delta kx}{2} - \frac{\Delta \omega t}{2} \right) \sin \left( \frac{(2k + \Delta k)x - (2\omega + \Delta \omega)t}{2} \right) \]

Substituting, \( 2k + \Delta k \approx 2k \) and \( \omega + \Delta \omega \approx 2\omega \)

\[ y = 2A \cos \left( \frac{\Delta kx}{2} - \frac{\Delta \omega t}{2} \right) \sin (kx - \omega t) \]

The above equation represents a sine wave of angular frequency \( \omega \) and wave number \( k \) whose amplitude is modulated with the angular frequency \( \Delta \omega / 2 \) and wave number \( \Delta k/2 \).

Group velocity is defined as the rate at which the amplitude is modulated or the rate at which energy is transported or the velocity of motion of the wave packet.

From eqn. (iii) the rate of modulation of amplitude, \( V_a = \Delta \omega / \Delta k = d\omega / dk \)

\[ d\omega = 2\pi d \nu = 2\pi d \left( \frac{E}{\hbar} \right) \quad \text{and} \quad dk = \frac{2\pi}{\hbar} = 2\pi hv \]

\[ \therefore \quad V_a = d\omega / dk = dE / dp \]
DAVISson Germer EXPERiment

The first experimental proof of the wave nature of electron was demonstrated in 1927 by two American physicists C.J Davison and L.H Germer. The basis of their experiment was that since the wavelength of an electron is of the order of spacing of atoms of a crystal, a beam of electrons shows diffraction effects when incident on a crystal.

The figure shows the experimental setup.

Electrons are emitted by thermionic emission from the electron gun A. They are passed between the anode and the cathode, which accelerates the electrons. These accelerated electrons were made to fall on a nickel crystal O normally. The beam of electrons is diffracted by the crystal and received at an angle φ by a detector. The intensity of the diffracted electrons is measured by the detector as a function of angle φ and also the scattered electron current.

The polar plot was plotted between the intensity of scattered electrons and the angle φ at various accelerating voltages.

The kink at 54V gives the evidence of electron waves, since a strong diffraction is observed at φ=50° for 54V. The Nickel crystal acts like a diffraction grating with spacing ‘d’.

We know from Bragg’s Law $2dsin \theta =n\lambda$

In this experiment

\[ d = 0.91\text{Å (Nickel)}, \]
\[ \theta = 65°(when \phi=50°), \]
\[ n=1 \]

Using Bragg’s Law for the above values we get

\[ \lambda = 1.65\text{Å}. \]
Now if we use the de-broglie’s theoretical formula to find the De-broglie wavelength associated with the electrons accelerated through a potential $V$ (non-relativistic):

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2meV}}$$

$$\lambda = \frac{6.63 \times 10^{-34}}{\sqrt{2 \times 9.1 \times 10^{-31} \times 1.6 \times 10^{-19} \times 54}}$$

$$\lambda = 1.66 \text{ Å}$$

Hence, the De Broglie wavelength of electron waves determined Davisson Germer experiment and those calculated from De BROGLIE hypothesis are found to be in close agreement. Thus, the Davisson Germer experiment confirms the de-broglie hypothesis of matter waves.

**HEISENBERG UNCERTAINTY PRINCIPLE**

HUP states that it is not possible to determine simultaneously and with unlimited precision, a pair of conjugate variables like position and momentum of a particle.

$$\Delta x \Delta p \geq \frac{\hbar}{4\pi}$$

Heisenberg’s Uncertainty Principle states that there is inherent uncertainty in the act of measuring a variable of a particle. Commonly applied to the position and momentum of a particle, the principle states that the more precisely the position is known the more uncertain the momentum is and vice versa. This is contrary to classical Newtonian physics which holds all variables of particles to be measurable to an arbitrary uncertainty given good enough equipment.

It is hard to imagine not being able to know exactly where a particle is at a given moment. It seems intuitive that if a particle exists in space, then we can point to where it is; however, the Heisenberg Uncertainty Principle clearly shows otherwise. This is because of the wave-like nature of a particle. A particle is spread out over space so that there simply is not a precise location that it occupies, but instead occupies a range of positions. Similarly, the momentum cannot be precisely known since a particle consists of a packet of waves, each of which have their own momentum so that at best it can be said that a particle has a range of momentum.
APPLICATIONS OF HEISENBERG UNCERTAINTY PRINCIPLE

1. The non-existence of free electron in the nucleus.

The diameter of nucleus of any atom is of the order of $10^{-14}$ m. If any electron is confined within the nucleus then the maximum uncertainty in its position ($\Delta x$) must not be greater than $10^{-14}$ m. According to Heisenberg’s uncertainty principle

$$\Delta x \Delta p \geq \frac{h}{2\pi}$$

Therefore, the minimum uncertainty in momentum corresponding to maximum uncertainty in position

$$\Delta p \geq \frac{h}{2\pi \Delta x} = \frac{6.63 \times 10^{-34}}{2 \times 3.14 \times 10^{-14}}$$

$$\Delta p \geq 1.055 \times 10^{-20} \text{ kg m/s}$$

If this is the minimum uncertainty in the momentum of electron and then the momentum of the electron must be at least of the same order of magnitude i.e,

$$p = \Delta p$$

According to the theory of relativity the energy of a particle is given by

$$E = mc^2 = \left( m_0 c^2 \right) / \left( 1-v^2/c^2 \right)^{1/2}$$

Where

$m_0$ is the particle’s rest mass and $m$ is the mass of the particle with velocity $v$.

Squaring the above equation we get,

$$E^2 = \left( m_0^2 c^4 \right) / \left( 1-v^2/c^2 \right) = \left( m_0^2 c^6 \right) / \left( c^2 - v^2 \right)$$

Momentum of the particle is given by $p = mv = \left( m_0 v \right) / \left( 1-v^2/c^2 \right)^{1/2}$
And \( p^2 = \frac{m_0 v^2 c^2}{1 - \frac{v^2}{c^2}} \)

then \( p^2 c^2 = \frac{m_0 v^2 c^4}{(c^2 - v^2)} \)

\[ E^2 - p^2 c^2 = \frac{(m_0 c^4)}{(c^2 - v^2)} \]

or \( E^2 = \frac{p^2 c^2 + m_0 c^4}{c^2 - v^2} \)

Substituting the value of momentum \( p = 1.055 \times 10^{-20} \text{ kg m/s} \) and the rest mass as \( m_0 = 9.1 \times 10^{-31} \text{ kg} \)

we get the kinetic energy of the electron as

\[ E^2 \geq (3 \times 10^8)^2 (0.25 \times 10^{-40} + 7.469 \times 10^{-44}) \]

The second term in the above equation being very small and may be neglected then we get

\[ E \geq 1.5 \times 10^{-12} \text{ J} \quad \text{OR} \]

\[ E \geq 9.4 \text{ MeV} \]

The above value for the kinetic energy indicates that an electron with a momentum of \( 1.055 \times 10^{-20} \text{ kg m/s} \) and mass of \( 9.1 \times 10^{-31} \text{ kg} \) to exist within the nucleus it must have energy equal to or greater than this value. But the experimental results on β decay show that the maximum kinetic an electron can have when it is confined within the nucleus is of the order of 3 - 4 Mev. Therefore the electrons cannot exist within the nucleus.

2. **Radius of Bohr’s first orbit**

If \( \Delta x \) and \( \Delta p_x \) are the uncertainties in the simultaneous measurements of position and momentum of the electron in the first orbit of radius ‘\( r \)’, then from uncertainty principle

\[ \Delta x \Delta p \geq \frac{h}{2\pi} \]

The minimum uncertainty in momentum corresponding to maximum uncertainty in position is given:

\[ \Delta p \geq \frac{h}{2\pi \Delta x} = \frac{h}{2\pi r} \]

\[ K.E = \frac{p^2}{2m} \]

\[ P.E = -\frac{Z e^2}{4\pi \varepsilon_0 r} \]

Total energy, \( T = K.E + P.E \)

Now \( p = \Delta p \) (min uncertainty in momentum) and \( \Delta x = r \) (max uncertainty in position)

\[ Uncertainty \ in \ total \ energy \ , \ \Delta E = \frac{h^2}{4\pi^2 r^2 2m} - \frac{Z e^2}{4\pi \varepsilon_0} \quad (3) \]

The Uncertainty in total energy will be minimum if
Therefore equation (3) represents the condition of minimum in the first orbit.

For Hydrogen $Z = 1$. Hence, the radius of first orbit is given by

$$r = 0.53\text{Å}$$

Using $r = 0.53\text{Å}$ in eq (3) we find that $d^2 \Delta E/dr^2$ comes out to be positive.


dr \frac{d(\Delta E)}{dr} = 0 \text{ we get } \Rightarrow r = \frac{\hbar^2 \epsilon_0}{m\epsilon e^2} \quad (4)

Therefore, with the help of Heisenberg’s uncertainty principle, one can determine the radius of the Bohr’s first orbit.

**WAVE FUNCTION AND ITS SIGNIFICANCE**

Matter waves are represented by a complex function, $\Psi(x,t)$, which is called wave function. The wave function is not directly associated with any physical quantity but the square of the wave function represents the probability density in a given region. The wave function should satisfy the following conditions:

(i) $\Psi$ should be finite
(ii) $\Psi$ should be single valued
(iii) $\Psi$ and its first derivative should be continuous
(iv) $\Psi$ should be normalizable.

$$\int \int \Psi(r)^* \Psi(r) dr = 1$$

**SCHRODINGER’S WAVE EQUATION**

**TIME INDEPENDENT SCHRODINGER’S EQUATION**

The classical wave equation that describes any type of wave motion can be given as

$$\frac{\partial^2 y(x,t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 y(x,t)}{\partial t^2}$$
Where \( y \) is a variable quantity that propagates in ‘x’ direction with a velocity ‘v’. Matter waves should also satisfy a similar equation and we can write the equation for matter waves as:

\[
\frac{\partial^2 \Psi(x, t)}{\partial x^2} = \frac{1}{v^2} \frac{\partial^2 \Psi(x, t)}{\partial t^2} \quad \cdots (i)
\]

We can eliminate the time dependence from the above equation by assuming a suitable form of the wave function and making appropriate substitutions.

\[
\Psi(x, t) = \Psi_0(x) e^{-i\omega t} \quad \cdots (ii)
\]

Differentiate eqn. (ii) w.r.t. ‘x’ successively to obtain

\[
\frac{\partial \Psi(x, t)}{\partial x} = d\Psi_0(x) e^{-i\omega t} \quad \text{and} \quad \frac{\partial^2 \Psi(x, t)}{\partial x^2} = \frac{d^2 \Psi_0(x)}{dx^2} e^{-i\omega t} \quad \cdots (iii)
\]

Differentiate eqn. (ii) w.r.t. ‘x’ successively to obtain

\[
\frac{\partial^2 \Psi(x, t)}{\partial t^2} = (-i\omega)^2 \Psi_0(x) e^{-i\omega t} \quad \cdots (iv)
\]

Substituting (iii) and (iv) in eqn. (i), we obtain

\[
\frac{d^2 \Psi_0(x)}{dx^2} = -\frac{\omega^2}{v^2} \Psi_0(x) = -k^2 \Psi_0(x) \quad \cdots (vi)
\]

or

\[
\frac{d^2 \Psi_0(x)}{dx^2} + k^2 \Psi_0(x) = 0 \quad \cdots (vii)
\]

\[
k = \frac{2\pi}{\lambda} \quad \text{and} \quad \lambda = \frac{h}{p}, \quad \therefore k^2 = \frac{4\pi^2 p^2}{h^2}
\]

The energy of the particle, ‘E’ is the sum of kinetic and potential energy

\[
E = \frac{p^2}{2m} + V \quad \text{or} \quad p^2 = 2m(E - V) \quad \text{or} \quad k^2 = 8\pi^2 m(E - V)/h^2
\]

Substituting the value of \( k^2 \) and replacing \( \Psi_0(x) \) by \( \Psi(x) \) in eqn.(vii), we obtain the Schrodinger’s time independent wave equation in one dimension as:

\[
\frac{d^2 \Psi(x)}{dx^2} + \frac{8\pi^2 m(E - V)}{h^2} \Psi(x) = 0
\]

The above equation can be solved for different cases to obtain the allowed values of energy and the allowed wave functions.
**TIME DEPENDENT SCHRODINGER’S EQUATION**

In the discussion of the particle in an infinite potential well, it was observed that the wave function of a particle of fixed energy \( E \) could most naturally be written as a linear combination of wave functions of the form

\[
\Psi(x, t) = A e^{i(kx - \omega t)}
\]

representing a wave travelling in the positive \( x \) direction, and a corresponding wave travelling in the opposite direction, so giving rise to a standing wave, this being necessary in order to satisfy the boundary conditions. This corresponds intuitively to our classical notion of a particle bouncing back and forth between the walls of the potential well, which suggests that we adopt the wave function above as being the appropriate wave function for a free particle of momentum \( p = \hbar k \) and energy \( E = \hbar \omega \). With this in mind, we can note that

\[
\frac{\partial^2 \Psi}{\partial x^2} = -k^2 \Psi
\]

which can be written, using \( E = p^2 / 2m = \hbar^2 k^2 / 2m \):

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \Psi}{\partial x^2} = \frac{p^2}{2m} \Psi.
\]

Similarly

\[
\frac{\partial \Psi}{\partial t} = -i \omega \Psi
\]

which can be written, using \( E = \hbar \omega \):

\[
\hbar \frac{\partial \Psi}{\partial t} = \hbar \omega \Psi = E \Psi.
\]

then note that

We now generalize this to the situation in which there is both a kinetic energy and a potential energy present, then \( E = p^2 / 2m + V(x) \) so that

\[
E \Psi = \frac{p^2}{2m} \Psi + V(x) \Psi
\]

where \( \Psi \) is now the wave function of a particle moving in the presence of a potential \( V(x) \). But if we assume that the results above still apply in this case then we have

\[
-\frac{\hbar^2}{2m} \frac{\partial^2 \psi}{\partial x^2} + V(x) \Psi = i \hbar \frac{\partial \psi}{\partial t}
\]

which is the famous time dependent Schrodinger wave equation. It is setting up and solving this equation, then analyzing the physical contents of its solutions that form the basis of that branch of quantum mechanics known as wave mechanics. Even though this equation does not look like the familiar wave equation that describes, for instance, waves on a stretched string, it is nevertheless referred to as a ‘wave equation’ as it can have solutions that represent waves propagating through space. We have seen an example of this: the harmonic wave function for a free particle of energy \( E \) and momentum \( p \), i.e. is a solution of this equation with, as appropriate for a free particle, \( V \)
\( (x) = 0 \). But this equation can have distinctly non-wave like solutions whose form depends, amongst other things, on the nature of the potential \( V(x) \) experienced by the particle.

\[
\Psi(x, t) = Ae^{-i(px-Et)/\hbar}
\]

In general, the solutions to the time dependent Schrödinger equation will describe the dynamical behaviour of the particle, in some sense similar to the way that Newton’s equation \( F = ma \) describes the dynamics of a particle in classical physics. However, there is an important difference. By solving Newton’s equation we can determine the position of a particle as a function of time, whereas by solving Schrodinger’s equation, what we get is a wave function \( \Psi(x, t) \) which tells us (after we square the wave function) how the probability of finding the particle in some region in space varies as a function of time.

**PARTICLE IN AN INFINITE POTENTIAL BOX**

Consider a particle of mass ‘m’ confined to a one dimensional potential well of dimension ‘L’. The potential energy, \( V = \infty \) at \( x=0 \) and \( x=L \) and \( V = 0 \) for \( 0 \leq x \leq L \). The walls are perfectly rigid and the probability of finding the particle is zero at the walls. Hence, the boundary conditions are, \( \Psi(x) = 0 \) at \( x=0 \) and \( x=L \). The one dimensional time independent wave equation is:

\[
\frac{d^2\Psi(x)}{dx^2} + \frac{8\pi^2 m(E-V)}{\hbar^2} \Psi(x) = 0
\]

We can substitute \( V=0 \) since the particle is free to move inside the potential well

\[
\frac{d^2\Psi(x)}{dx^2} + \frac{8\pi^2 mE}{\hbar^2} \Psi(x) = 0
\]

Let \( \frac{8\pi^2 mE}{\hbar^2} = k^2 \) and the wave equation is,

\[
\frac{d^2\Psi(x)}{dx^2} + k^2\Psi(x) = 0
\]

The above equation represents simple harmonic motion and the general solution is

\[
\Psi(x) = A \sin kx + B \cos kx
\]

Applying the condition, \( \Psi(x) = 0 \) at \( x = 0 \), we get \( 0 = A.0 + B \) which implies that \( B = 0 \)

Substituting the condition \( \Psi(x) = 0 \) at \( x = L \), we get, \( A \sin kL = 0 \)

\( kL = n\pi \) or, \( k = n\pi/L \) where \( n = 1,2,3, \ldots \) is a positive integer.

**Eigen values of energy** :

\[
E_n = \frac{k^2\hbar^2}{8\pi^2 m} = \frac{n^2\pi^2\hbar^2}{8\pi^2 mL^2} = n^2\hbar^2/8mL^2
\]
‘n’ is the quantum number corresponding to a given energy level. n=1 corresponds to the ground state, n=2 corresponds to the first excited state and so on.

**Eigen functions**: The allowed wavefunctions are

\[ \Psi_n(x) = A \sin\left(\frac{n\pi x}{L}\right) \]

The constant A can be evaluated from normalization condition

\[ \int_{x=0}^{x=L} A^2 \sin^2\left(\frac{n\pi x}{L}\right) = 1 \]

\[ A = \sqrt{\frac{2}{L}} \]

hence, the eigen functions are \( \Psi_n(x) = \sqrt{\frac{2}{L}} \sin\left(\frac{n\pi x}{L}\right) \)

Graphical representation of \( E_n, P_n(x) \) and \( \Psi_n(x) \) for \( n = 1, 2 \) and 3 quantum states
INTERFERENCE
Coherent Sources

It is found that it is not possible to show interference due to two independent sources of light because a large number of difficulties are involved. Two sources may emit light wave of different amplitude and wavelength and the phase difference between the two may change with time. The fundamental requirement to get a well-defined interference pattern is that the phase difference between the two waves should be constant. The two sources are said to be coherent if they emit light waves of same frequency nearly the same amplitude and are always in phase with each other. In actual practice, it is not possible to have two independent sources which are coherent. But for experimental purposes, two virtual sources formed from a single source can act as coherent sources. Methods for producing coherent sources are divided into following two parts have been divided (i) interference of light takes place between waves from two sources formed due to single source (division of wavefront) (ii) interference takes place between the waves from the real source and virtual source (division of amplitude).

Interference in Thin Films

The film of transparent material like a drop of oil spread on the surface of water, show brilliant colours when exposed to an extended source of light. This phenomenon can be explained on interference basis. Here interference takes place between rays reflected from the upper and from the lower surface of the film.

**Case I: Thin film of uniform thickness:**

(a) Reflection Pattern: Let us consider a thin film of thickness \( t \), refractive index \( \mu \) and a ray \( AB \) of monochromatic light of \( \lambda \) is falling on it at an angle \( i \). This ray is partly reflected and partly refracted as shown in Figure.

Path difference \( = (BD + DE)_{\text{film}} - (BH)_{\text{air}} \)

\[ = [\mu (BD + DE) - BH]_{\text{air}} \]

\[ = [\mu (BD + DG + GE) - BH] \]

\[ = \mu (BD + DG) \]

As \( \mu GE = BH \), \( BH = EB \sin i \)

\( GE = EB \sin r \), \( \mu GE = EB = \sin i \)

Hence \( \mu = \frac{\sin i}{\sin r} \)
Extend BL and ED to meet at K we have

$LKD = LNDE = r$

Triangles BLD and LKD are congruent

$BD = DK$

Then path difference $= \mu(BD + DG) = \mu(KD + DG)$

$= \mu KG = \mu KB \cos r$

$\Delta = 2 \mu t \cos r$

Now, here since reflection from denser medium is taking place by one ray so additional path difference of $\lambda/2$ is taken into account. Thus the total path difference is

$\Delta_T = 2 \mu t \cos r - \lambda/2$

**Condition for maxima**

$\Delta_T = (2 \mu \cos r - \lambda/2) = n \lambda$

$2 \mu t \cos r = (2n + 1) \lambda/2$ \quad $n = 0, 1, 2, 3, \ldots$

**Condition for minima**

$\Delta_T = (2 \mu t \cos r - \lambda/2) = (2n + 1) \lambda/2$

$2 \mu t \cos r = (n + 1) \lambda$

$n$ is the integer so $(n + 1)$ is also integer and can be taken as $n$

$2 \mu t \cos r = n \lambda$ \quad $n = 0, 1, 2$

when $\cos r$ is kept constant and thickness increases gradually, the path difference becomes $\lambda/2, \lambda, 3 \lambda/2, 2 \lambda, 5 \lambda/2$, etc. and as a result the film will appear dark $(t = 0)$, bright, dark and so on. On the other hand if $t$ is constant and $r$ is varied we again get a series of maxima and minima.

**(b) Transmitted Pattern:** Here the path difference two rays OEMS and PR will be

$\Delta = \mu (DE + EM) - DH$
\[ = 2 \mu t \cos r \ldots \]

In this case there will be no additional path difference so the total path difference

\[ \Delta_T = 2 \mu t \cos r \]

Condition for maxima

\[ \Delta = 2 \mu t \cos r = n\lambda \quad n = 0, 1, 2, 3, \ldots \]

Condition for minima

\[ \Delta = 2 \mu t \cos r = (2n + 1) \lambda / 2 \quad n = 0, 1, 2, 3, \ldots \]

We find that the conditions for maxima and minima are found in case of transmitted pattern are opposite to those found in case of reflected pattern. Under the same conditions of the film looks dark in reflected pattern it will look bright in transmitted pattern.

**Colours of thin films:** When white light is incident on a thin film only few wavelengths will satisfy the condition of maxima and therefore corresponding colours will seen in the pattern. For other wavelengths condition of minima is satisfied, and so corresponding colours will be missing in the pattern. The coloration of film vary with t and r. Therefore if one vary either t or r a different set of colours will be observed. Since the condition for maxima and minima are opposite in case of reflected and transmitted pattern. So the colours found in two patterns will be complimentary to each other.

**Necessity of broad source:** When point source is used only a small portion of the film can be seen through eye and as a result the whole interference pattern cannot be seen. But when a broad source is used rays of light are incident at different angles and reflected parallel beam reach the eye and whole beam and complete pattern is visible.

**Case I: Thin film of non-uniform thickness** (Wedge shaped thin film):

the wedge shaped film as shown in Figure 4.5. Let a ray from S is falling on the film and after deflections produce interference pattern.

\[ \Delta = [(PF + FE)_{film} + (PK)_{air}] \]

\[ = \mu (PF + FE) - (PK) \]
= μ (PN + NF + FE) – PK
= μ (PN + NF + FE – PN)
= μ (NF + FE)
= μ (NF + FL)
= μ (NL) = 2 μ t cos (r + θ)

Then total path difference considering refraction from denser medium is taking place

\[ \Delta_T = 2 \mu t \cos (r + \theta) - \lambda/2 \]

**Condition for maxima**

\[ 2\mu \cos (r + \theta) = (2n + 1) \lambda/2 \]
\[ n = 0, 1, 2, \ldots \ldots \]

**Condition for minima**

\[ 2\mu \cos (r + \theta) = n\lambda \]
\[ n = 0, 1, 2, 3, \ldots \ldots \ldots \]

Thus the film will appear bright if the thickness \( t \) satisfies the condition of maxima

\[ t = (2n + 1) \lambda / 4\mu \cos (r + \theta) \]

and it will appear dark when

\[ t = n \lambda / 2\mu \tan \theta \cos (r + \theta) \]

Hence, we move along the direction of increasing thickness we observe dark, bright, dark \( \lambda \), \ldots fringes.

For \( t = 0 \) i.e., at the edge of film \( \Delta = \lambda/2 \) so the film will appear dark. Then width of the fringes so observed can be found

\[ \beta = n \lambda / 2\mu \tan \theta \cos (r + \theta) \]

In case of normal incidence \( r = 0 \)

\[ 2 \mu t = (2n + 1) \lambda/2 \text{ (maxima)} \]

\[ 2 \mu t = n \lambda \text{ (minima)} \]
\[ \beta = \frac{\lambda}{2\mu\theta} = \text{fringe width} \]

**Newton’s Ring Experiment**

We have seen that thin film causes a path difference \(2\mu t \cos r\) between interfering rays. Now, when area of the film is small, the rays from various portions of the film reaching the eye have almost the same inclination. So it is the variation in thickness that gives fringes. Each fringe is the locus of all such points where thickness is same. Such fringes are known as Newton’s fringes or fringes of equal thickness. On the other hand when film has uniform thickness the path difference changes with \(r\) only. Each fringe in this case represents the locus of all points on the film, from which rays are equally inclined to the normal. Such fringes are called Haidinger’s fringes or fringes of equal inclination. To get such fringes the source must be an extended one.

**Newton’s rings:** It is a special case of interference in a film of variable thickness such as that formed between a plane glass plate and a convex lens in contact with it. When monochromatic light falls over it normally we get a central dark spot surrounded by alternatively bright and dark circular rings. When white light is used the rings would be coloured.

**Experimental Arrangement:** Let \(S\) be the extended source of light, rays from which after passing through a lens \(L\) falls upon a glass plate \(G\) at 45°. After partial reflection these rays fall on a plano convex lens \(P\) placed on the glass plate \(E\). The interference occurs between the rays reflected from the two surfaces of the air film and viewed through microscope \(M\) as shown Figure.

**Theory:** The air film formed is of wedge shape so the path difference produced will be

\[ \Delta = 2\mu t \cos(r+\theta) - \frac{\lambda}{2}, \quad \text{For normal incident } r = 0 \]

So \[ \Delta = 2\mu t - \frac{\lambda}{2} \]

At the point of contact \(t = 0\)

So \[ \mu = \frac{\lambda}{2} \], The central fringe will be dark.

**Condition for maxima and minima**

\[ 2\mu t = (2n + 1) \frac{\lambda}{2} \]

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2 µt = nλ  

we get alternatively bright and dark rings.

**Diameter of the rings:**

NP x NQ = ND x NO

NP= NQ= r, radius of ring under consideration.

Then  
\[ r^2 = t (2R - t) = 2Rt - t^2 = 2Rt \]

\[ t = r^2/2R \]

**Diameter of bright rings \( D_n \):** From the condition of maxima

\[ 2 \mu \frac{r^2 n}{2R} = (2n + 1) \frac{\lambda}{2} , \]

\[ r^2 n = (2n + 1) \frac{\lambda R}{2 \mu} = (D_n / 2)^2 \]

\[ D_n^2 = 2 (2n + 1) \frac{\lambda R}{\mu} \]

\[ = 2 (2n + 1) \frac{\lambda R}{\mu} \]  

(For air, \( \mu = 1 \))

\[ D_n = \sqrt{(2A)} \sqrt{(2n + 1)} = K \sqrt{(2n + 1)} \]

\[ D_n = \sqrt{\text{odd number}} . \]

Thus, the diameter of bright rings are proportional to the square root of the odd natural numbers.

**Diameter of dark ring \( D_n \):** Using the condition for minimum

\[ 2\mu \frac{r_n^2}{2R} = n\lambda \]

which gives

\[ (D_n)^2 = 4n\lambda R \]

Or  
\[ D_n = \sqrt{(4 \lambda R)} \sqrt{n} \]

\[ D_n = \sqrt{n} \]
Thus, the diameter of dark rings are proportional to the square root of natural numbers.

**Application of Newton’s Ring**

**Measurement of Wavelength of light by Newton’s Rings**

For dark rings we know

\[(D_n)^2/n = 4\lambda R\]

\[\lambda = (D_n)^2/n / 4nR\]

But as the fringes around the centre are not very clear so \(n\) cannot be measured correctly.

To avoid any mistake one can consider two clear fringes \(n\)th and \((n + p)\)th

So,

\[(D_n)^2 / n = 4\lambda R\quad (D_{n+p})^2 = 4(n+p) \lambda R\]

Then

\[((D_{n+p})^2 - (D_n)^2) = 4(n+p) \lambda R\]

Or

\[\lambda = ((D_{n+p})^2 - (D_n)^2) / 4pR\]

Thus, measuring the diameters and knowing \(p\) and \(R\), \(\lambda\) can be measured.

**Measurement of Refractive Index of Liquid by Newton’s Rings**

For this purpose liquid film is formed between the lens and glass plate.

We have, as above,

which give

\[[(D_{n+p})^2 - (D_n)^2]_{\text{liquid}} = 4p\lambda R/\mu\]

\[[(D_{n+p})^2 - (D_n)^2]_{\text{air}} = 4p\lambda R\]

One can see that rings contract with the introduction of liquid.

**DIFFRACTION**

Diffraction phenomenon can be classified into following two classes only

on the of positions of source and screen:
(i) Fresnel’s diffraction: In this class either the source or screen or both are at distance from the obstacle and thus distances are important. Here the incident wavefronts are either spherical or cylindrical.

(ii) Frounhofer’s diffraction: In this class both the source and the screen are at distance from the obstacle and thus inclination are important not the distances the wavefront is plane one.

Fraunhofer diffraction at single Slit:

Let S is a source of monochromatic light of wavelength ‘A, L is collimating lens AB is a slit of width a, L’ is another converging lens and XY is the screen light coming out from source and passing through slit is focused at the screen. A diffraction pattern is obtained on the screen which consists of central bright band having alternate dark and bright bands of decreasing intensity on both the sides. The complete arrangement is shown in Figure.

Analysis and explanation: According to Huygen’s theory a point in AB send out secondary waves in all directions. The diffracted ray along the direction of incident ray are focussed at C and those at an angle ε and focussed at P and P’. Being at equidistant from all slits points, secondary wave will reach in same phase at C and so the intensity well be maximum. For the intensity at P, let AN is normal to BN, then path difference between the extreme rays is

$$\Delta = BN = AN \sin \theta = a \sin \theta = 2\pi/\lambda \cdot a \sin \theta$$

which is zero for the ray from A and maximum for the ray from B. Let AB consists of n secondary sources then the phase difference between any two consecutive source will be

$$2\pi/\lambda \cdot a \sin \theta = \delta \text{ (say)}$$

The resultant amplitude and phase at P will be

$$R = a \sin n\delta/2 /\sin \delta/2$$

$$= a \sin \pi a \sin \theta / \lambda / \sin \pi a \sin \theta /n\lambda$$

$$= a \sin \alpha / \sin \alpha/n = na \sin \alpha/\alpha = A \sin \alpha / \alpha$$

A = n α and α = πasinθ / λ

Corresponding the intensity is

$$I = R^2 = A^2 \sin^2 \alpha / \alpha^2$$
Condition of maxima and minima

\[ \frac{dI}{d\alpha} = 0 \Rightarrow \frac{d}{d\alpha} \left[ A^2 \sin^2 \alpha / \alpha^2 \right] = 0 \]

where (i) \( \sin \alpha / \alpha = 0 \) or (ii) \( \alpha = \tan \alpha \)

Condition of minimum intensity

Intensity will be 0 when \( \sin \alpha / \alpha = 0 \) or \( \sin \alpha = 0 \)

\[ \alpha = m\pi \quad \text{or} \quad \pi \alpha \sin \theta / \lambda = m\pi \]

\[ a \sin \theta = m\lambda \]

**Condition of maximum intensity:**

Intensity will be maximum when \( \alpha = \tan \alpha \)

The value of \( \alpha \) satisfying this equation are obtained graphically by plotting the curve \( y = \alpha \) and \( y = \tan \alpha \) on the same graph (Figure). The point of intersection will give

\[ \alpha = 0, \pm 3\pi/2, \pm 5\pi/2, \pm 7\pi/2, \]

\[ \pm 0, \pm 1.43\pi, \pm 2.462\pi, \pm 3.471\pi, \]

Graphical representation of positions of secondary maxima’s in the diffraction

\( \alpha = 0 \) correspond point to central maximum whose intensity is given as

\[ I = L \quad A^2 \left[ \sin^2 \alpha / \alpha^2 \right] = A^2 = I_0 \]

The other maxima are given by

\( A \sin \theta = (2m + 1) \lambda / 2 \) and their intensities as

\[ m = 1: \quad I_1 = A^2 \left( \sin 3\pi/2 \right)^2 = 4I_0 / 9\pi^2 = I_0 / 22 \]

\[ m = 2: \quad I_2 = 4I_0 / 25\pi^2 = I_0 / 61 \]

\[ m_3 = I_3 = 4I_0 / 49\pi^2 = 10/121 \text{ and so on} \]

The diffraction pattern consists of a bright central maximum surrounded alternatively by minima maximum.
Plane Transmission Diffraction Grating (N-Slits Diffraction/Diffraction due to double slits)

A plane diffraction grating is an arrangement consisting of a large number of close, parallel, straight, transparent and equidistant slits, each of equal width $a$, with neighboring slits being separated by an opaque region of width $b$. A grating is made by drawing a series of very fine, equidistant and parallel lines on an optically plane glass plate by means of a fine diamond pen. The light cannot pass through the lines drawn by diamond; while the spacing between the lines is transparent to the light. There can be 15,000 lines per inch or more is such a grating to produce a diffraction of visible light. The spacing $(a + b)$ between adjacent slits is called the diffraction element or grating element. If the lines are drawn on a silvered surface of the mirror (plane or concave) then light is reflected from the positions of mirrors in between any two lines and it forms a plane concave reflection grating. Since the original gratings are quite expensive for practical purposes their photographic reproductions are generally used.

The commercial gratings are produced by taking the cast of an actual grating on a transparent film such as cellulose acetate. A thin layer of collodin solution (celluloid dissolved in a volatile solvent) is poured on the surface of ruled grating and allowed to dry. Thin collodin film is stripped off from grating surface. This film, which retains the impressions of the original grating, is preserved by mounting the film between two glass sheets. Now-a-days holographic gratings are also produced. Holographic gratings have a much large number of lines per cm than a ruled grating

Theory of Grating: Suppose a plane diffraction grating, consisting of large number of $N$ parallel slits each of width $a$ and separation $b$, is illuminated normally by a plane wave front of monochromatic light of wavelength $\lambda$. as shown in Figure 5.8. The light diffracted through $N$ slits is focused by a convex lens on screen $XY$ placed in the focal plane of the lens $L$. The diffraction pattern obtained on the screen with very large number of slit consists of extremely sharp principle interference maximum; while the intensity of secondary maxima becomes negligibly small so that these are not visible in the diffraction pattern. Thus, if we increase the number of slits ($N$), the intensity of principal maxima increases. The direction of principal maxima are given by

$$\sin \beta = 0, \text{ i.e., } \beta = \pm n\pi, \text{ where } n = 0, 1, 2, 3, \ldots$$

$$\frac{\pi}{\lambda} (a + b) \sin \theta = \pm n \lambda \Rightarrow (a + b) \sin \theta = \pm n \lambda. \quad \ldots \ (1)$$

If we put $n = 0$ in equation (1), we get $\theta = 0$ and equation (1) gives the direction of zero order principal maximum. The first, second, third, … order principal maxima may be obtained by putting $n = 1, 2, 3, \ldots$ in equation (1).

Minima: The intensity is minimum, when

$$\sin N\beta = 0; \text{ but } \sin \beta \neq 0$$

Therefore $N\beta = \pm m\pi$
Engineering Physics

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\[ N \frac{\pi}{\lambda} (a + b) \sin \theta = \pm m\pi \]

\[ N (a + b) \sin \theta = \pm m\lambda \quad (2) \]

Here can have all integral values except 0, N, 2N, 3N, … because for these values of m, \( \sin 13 = 0 \) which gives the positions of principal maxima. Positive and negative signs shows that the minima lie symmetrically on both sides of the central principal maximum. It is clear from equation (2) that for \( m = 0 \), we get zero order principal maximum, \( m = 1, 2, 3, 4, = (N - 1) \) gives minima governed by equation (2) and then at \( m = N \), we get principal maxima of first order. This indicates that, there are \( (N - 1) \) equispaced minima between zero and first orders maxima. Thus, there are \( (N – 1) \) minimum between two successive principal maxima.

**Secondary Maxima:** The above study reveals that there are \( (N- 1) \) minima between two successive principal maxima. Hence there are \( (N -2) \) other maxima coming alternatively with the minima between two successive principal maxima. These maxima are called secondary maxima. To find the positions of the secondary maxima, we first differentiate equation with respect to \( \beta \) and equating to zero

\[
\frac{dI}{d\beta} = A^2 \sin^2 \frac{\alpha}{\lambda^2} \cdot 2 \left[ \sin N\beta/\sin \beta \right] N \cos N\beta \sin \beta - \sin N \cos \beta/\sin 2\beta = 0
\]

\[ N \cos N\beta \sin \beta = \sin N\beta \cos \beta = 0 \]

\[ \tan N\beta = N \tan \beta \]

To find the intensity of secondary maximum, we make these of the triangle shown in Figure. We have \( N\beta = N \tan \beta/\sqrt{(1+N^2 \tan^2 \beta)} \)

**Therefore** \( \sin^2 \beta/\sin^2 \beta = (n^2 \tan^2 \beta)/\sin^2 \beta(1+n^2 \tan^2 \beta) \)

\[
\sin^2 N\beta/\sin^2 \beta = (n^2 \tan^2 \beta(1+N \tan^2 \beta))/\sin^2 \beta = N^2(1+n^2 \sin^2 \beta)
\]

Putting this value of \( \sin^2 N\beta/\sin^2 \beta \) we get

\[ I_S = A^2 \sin^2 \frac{\alpha}{\lambda^2} = N^2/[1+(N^2-1) \sin^2 \beta] \]

This indicates the intensity of secondary maxima is proportional to \( N^2 /[1+(N^2-1) \sin 2 \beta] \)

whereas the intensity of principal maxima is proportional to \( N^2 \).

**Absent Spectra with a Diffraction Grating**

It may be possible that while the first order spectra is clearly visible, second order may be not be visible at all and the third order may again be visible. It happen when for again angle of diffraction \( 0 \), the path difference between the diffracted ray from the two extreme ends of one slit is equal to an integral multiple of \( A \) if the path difference between the secondary waves from the corresponding point in the two halves
will be A/2 and they will cancel all one another effect resulting is zero intensity. Thus the mining of single slit pattern are obtained in the direction given by.

\[ a \sin \theta = m \lambda \]  

\[ a \sin \theta = m \lambda \]  

where \( m = 1, 2, 3, \ldots \) excluding zero but the condition for nth order principles maximum in the grating spectrum is \( (a + b) \sin \theta = n \lambda \)  

\[ (a + b) \sin \theta = n \lambda \]  

If the two conditions given by equation (2) are simultaneously satisfied then the direction in which the grating spectrum should give us a maximum every slit by itself will produce darkness in that direction and hence the most favourable phase for reinforcement will not be able to produce an illumination i.e., the resultant intensity will be zero and hence the absent spectrum. Therefore dividing equation (2) by equation (1)

\[ \frac{(a + b) \sin \theta}{a \sin \theta} = \frac{n}{m} \]

\[ \frac{(a + b)}{a} = \frac{n}{m} \]

This is the condition for the absent spectra in the diffraction pattern

If \( a = b \) i.e., the width of transparent portion is equal to the width of opaque portion then

from equation (3) \( n = 2m \) i.e., 2nd, 4th, 6th etc., orders of the spectra will be absent corresponds to the minima due to single slit given by \( m = 1, 2, 3 \) etc.

\[ b = 2a \& n = 3m \]

i.e., 3rd, 6th, 9th etc., order of the spectra will be absent corresponding to a minima due to a single slit given by \( m = 1, 2, 3 \) etc.

**Number of Orders of Spectra with a Grating**

The number of spectra that are visible in a given grating can be easily calculated with the help of the equation.

\[ (a + b) \sin \theta = n \lambda \]

\[ n = \frac{(a + b) \sin \theta}{\lambda} \]

Here \( (a + b) \) is the grating element and is equal to \( 1/N = 2.54 \) N cm, \( N \) being number of lines per inch in the grating. Maximum possible value of the angle of diffraction \( e \) is 90°, Therefore \( \sin \theta = 1 \) and the maximum possible order of spectra.
\[ N_{\text{max}} = \frac{(a+b)}{\lambda} \]

If \((a + b)\) is between \(\lambda\) and \(2\lambda\), i.e., grating element \((a + b) < 2\lambda\) then,

\[ n_{\text{max}} < 2\frac{\lambda}{\lambda} < 2 \]

and hence only the first order of spectrum is seen.

**Resolving Power of Optical Instruments**

When two objects are very close to each other, it may not be possible for our eye to see them separately. If we wish to see them separately, then we will have to make use of some optical instruments like microscope, telescope, grating, prism etc. The ability of an optical instrument to form distinctly separate images of two objects, very close to each other is called the resolving power of instrument. A lens system like microscope and telescope gives us a geometrical resolution while a grating or a prism gives a spectral resolution. In fact the image of a point object or line is not simply a point or line but what we get is a diffraction pattern of decreasing intensity. For a two point system two diffraction patterns are obtained which may and may not overlap depending upon their separation. The minimum separation between two objects that can be resolved by an optical instrument is called resolving limit of that instrument. The resolving power is inversely proportional to the resolving limit.

**Rayleigh Criterion of Resolution**

According to Lord Rayleigh’s arbitrary criterion two nearby images are said to be resolved if (i) the position of central maximum of one coincides with the first minima of the other or vice versa.

To illustrate this let us consider the diffraction patterns due to two wavelengths \(\lambda_1\) and \(\lambda_2\). There may be three possibilities. First let the difference \((\lambda_1 - \lambda_2)\) is sufficiently large so that central maximum are quite separate, this situation is called well resolved. Secondly consider that \((\lambda)\) is such that central maximum due to one falls on the first minima of the other. The resultant intensity curve shows a distinct dip in the
middle of two central maxima. This situation is called just resolved as the intensity of the dip can be resolved by our eyes.

\[ I_{\text{dip}} = 0.81 \ I_{\text{max}} \] … (1)

Thirdly let the \((\lambda_1 - \lambda_2)\) is very small such that they come still closes as shown in Figure. The intensity curves have sufficient overlapping and two images cannot be distinguished separately. The resultant curve almost appears as one maxima. This case is known as unsolved. Thus the minimum limit of resolution is that when two patterns are just resolved.

**Resolving Power of Plane Diffraction Grating**

We know that the diffraction grating has ability to produce spectrum i.e., to separate the lines of nearly equal wavelengths and therefore it has resolving capability. The resolving power of a grating may be defined as its ability to form separate diffraction maxima of two wavelengths which are very close to each other. If \(A.\) is the mean value of the two wavelengths and \(d\lambda\) is the difference between two then resolving power may be defined as resolving power = \(\lambda/d\lambda\).

**Expression for resolving power:** Let a beam of light having two wavelengths \(\lambda_1\) and \(\lambda_2\) is falling normally on a grating AB which has \((a + b)\) grating element and \(N\) number of slits as shown in Figure. After passing through grating rays forms the diffraction patterns which can be seen through telescope. Now, if these patterns are very close together they overlap and cannot be seen separately. However, if they satisfy the Rayleigh criterion, that is the wavelengths can be just resolved when central maxima due to one falls on the first minima of the other.

Let the direction of \(n^\text{th}\) principal maxima for wavelength \(A.1\) is given by

\[(a + b) \sin \theta_n = n\lambda_1\]

Or \(N (a + b) \sin \theta_n = Nn\lambda_1\)

and the first minima will be in the direction given by

\[N (a+b)\sin (\theta_n + d\theta_n) = m\lambda_1\]

where \(m\) is an integer except \(0, N, 2N \ldots\) because at these values condition of maxima will be satisfied. The first minima adjacent to the \(n^\text{th}\) maxima will be in the direction \((\theta_n + d\theta_n)\) only when \(m = (nN + 1)\). Thus

\[N (a + b) \sin (\theta_n + d\theta_n) = (nN + 1) \lambda_1\]

Therefore \((a + b) \sin (\theta_n + d\theta_n) = n\lambda_2\) or \(N (a + b) \sin (\theta_n + d\theta_n) = Nn\lambda_2\)
Now equating the two equations

\[(nN + 1) \lambda_1 = Nn. \lambda_2 \quad \text{or} \quad (nN + 1) \lambda = Nn (\lambda + d\lambda)\]

\[\lambda = Nn d\lambda.\]

\[\lambda_1 = \lambda, \lambda_2 - \lambda_1 = d\lambda, \lambda_2 = \lambda + d\lambda.\]

Thus resolving power of grating is found as

\[\text{R.P.} = \frac{\lambda}{d\lambda} = nN\]

Resolving power = order of spectrum x total number of lines on grating which can also be written as

\[N (a+b) \sin \theta \lambda = w \sin \theta_n / \lambda\]

where, \(m = N(a+b)\) is the total width of lined space in grating.

\[\text{R.P.}_{\text{MAX}} = n (a+b)/\lambda w/\lambda\]

\[\theta_n = 90^\circ\]

**Relation between Resolving Power and Dispersive Power of a Grating**

We know that resolving power

\[\text{R.P.} = \frac{\lambda}{d\lambda} nN\]

and dispersive power  \(D.p. d\theta / D\lambda = n / (a+b) \cos \theta_n\)

Therefore, \(\lambda / d\lambda = nN = N (a+b) \cos \theta_n n / (a+b) \cos \theta_n\)

\[\lambda / d\lambda = Ax d\theta / d\lambda\]

Resolving power = total aperture of telescope objective x dispersive power.

The resolving power of a grating can be increased by

(i) Increasing the number of lines on the grating \(N\).

(ii) Increasing the sides of spectrum \(n\).

(iii) Increasing the total width of grating ‘\(w\)’, for which one has to make use of whole aperture of telescopes objective.
FIBER OPTICS

Fundamental Ideas about Optical fibers and Propagation Mechanism and communication in optical fiber

Optical fiber is a wave guide used for optical communication. It is made of transparent dielectric materials whose function is to guide the light wave. An optical fiber consists of an inner cylindrical portion of glass, called core. The function of core is to carry the light from one end to another end by the principle of total internal reflection. The core is surrounded by another cylindrical covering called cladding. The refractive index of core is greater than the refractive index of cladding. Cladding helps to keep the light within the core. The propagation of light inside the optical fiber is shown in Fig. 1.

Let $\theta$ be the angle of incidence of the light ray with the axis and $r$ the angle of refraction. If $\theta$ be the angle at which the ray is incident on the fiber boundary, then $\theta = (90 - r)$. Let $n_1$, be the refractive index of the fiber. If $\theta > \theta_c$ critical angle where $\theta_c = \sin^{-1} \frac{n_2}{n_1}$ then the ray is totally internally reflected.

Acceptance angle, acceptance cone and numerical aperture.

The acceptance angle is the maximum angle from the fibre axis at which light may enter the fiber so that it will propagate in the core by total internal reflection. If a ray is rotated around the fibre axis keeping acceptance angle same, then it describes a conical surface as shown in Fig. Now only those rays which are coming into the fiber within this cone having a full angle 2 i will only be totally internally reflected and this confined within the fiber for propagations. Therefore this cone is called as acceptance cone.

Numerical aperture (NA) is a measure of the amount of light rays that can be accepted by the fiber and is more generally used term in optical fiber.

Consider a cylindrical optical fiber wire which consists of inner core of refractive index $n_1$ and an outer cladding of refractive index $n_2$ where $n_1 > n_2$. The typical propagations of light in optical fiber is shown in figure.

Now we will calculate the angle of incidence $i$ for which $\theta > \theta_c$ (critical angle) so that the light rebounds within the fiber.

Applying Snell’s law of refraction at entry point of the ray AB.
Where $n_0$ is the refractive index of medium from which the light enters in the fiber. From triangle BCE, 

$r = (90 - \theta)$

$\therefore \quad \sin r = \sin (90 - \theta)$

$\sin r = \cos \theta$ ................................................................. (2)

Substituting the value of $\sin r$ from Equation (2) in Equation (1), We get,

$n_0 \sin i = n_1 \cos \theta$

$\sin i = \left(\frac{n_1}{n_0}\right) \cos \theta$ ................................................................. (3)

If $i$ is increased beyond a limit, $\theta$ will drop below the critical value $\theta_c$ and the ray will escape from the side walls of the fiber. The largest value of $i$ which is $i_{\text{max}}$ occurs when $\theta = \theta_c$. Applying this condition in Equation (3),

$\sin i_{\text{max}} = \left(\frac{n_1}{n_0}\right) \cos \theta_c$ ................................................................. (4)

where $\theta_c = \sin^{-1} \frac{n_2}{n_1}$

$\therefore \quad \cos \theta_c = \sqrt{1 - \sin^2 \theta_c} = \frac{\sqrt{(n_1^2 - n_2^2)}}{n_1}$

From Equation (4), we have

$\sin i_{\text{max}} = \frac{n_1}{n_0} \sqrt{\frac{(n_1^2 - n_2^2)}{n_1}}$

$= \frac{\sqrt{(n_1^2 - n_2^2)}}{n_0}$ ................................................................. (5)

Almost all the time the ray is launched from air medium, then $n_0 = 1$ and $i_{\text{max}} = i$

$\sin i = \sqrt{(n_1^2 - n_2^2)}$

Where $i$ is called acceptance angle of the fiber.

$\therefore \quad i = \sin^{-1} \sqrt{(n_1^2 - n_2^2)}$

Hence the acceptance angle is defined as the maximum angle from the fiber axis at which light may enter the fiber so that it will propagate in the core by total internal reflection.

Now the light contained within the cone having a full angle $2i$ are accepted and transmitted through fiber. The cone associated with the angle $2i$ is called acceptance cone as shown in Fig.
Numerical aperture: Numerical aperture ‘NA’ determines the light gathering ability of the fiber. So it is a measure of the amount of light that can be accepted by the fiber. This is also defined as,

\[ NA = \sin i \]

\[ \therefore NA = \sqrt{n_1^2 - n_2^2} \] ................................. (6)

The NA may also be derived in terms of relative refractive index difference \( \Delta \) as,

\[ \Delta = \frac{n_1 - n_2}{n_1} = 1 - \frac{n_2}{n_1} \quad \text{Hence} \quad \frac{n_2}{n_1} = (1 - \Delta) \quad \text{Now From Equation (6)} \]

\[ \therefore NA = \sqrt{n_1^2 - n_2^2} = n_1\sqrt{1 - \left(\frac{n_2}{n_1}\right)^2} \]

Now substitute the value of \( \frac{n_2}{n_1} \) then after solving above equation will be

\[ \therefore NA = n_1\sqrt{2\Delta} \]

Types of optical fiber:
The different types of optical fibers are-
1. Step Index
   a. Single Mode
   b. Multimode
2. Guided Index

1. Step Index Fiber:
These types of fibers have sharp boundaries between the core and cladding, with clearly defined indices of refraction. The entire core uses single index of refraction.
Single mode fiber has a core diameter of 8 to 9 microns, which only allows one light path or mode.
**Multimode Step-Index Fiber:**
Multimode fiber has a core diameter of 50 or 62.5 microns (sometimes even larger). It allows several light paths or *modes*. This causes *modal dispersion* – some modes take longer to pass through the fiber than others because they travel a longer distance.

**2. Multimode graded Index Fiber:**
Graded-index refers to the fact that the refractive index of the core gradually decreases farther from the center of the core. The increased refraction in the center of the core slows the speed of some light rays, allowing all the light rays to reach the receiving end at approximately the same time, reducing dispersion.

As the above figure shows, the light rays no longer follow straight lines; they follow a serpentine path being gradually bent back toward the center by the continuously declining refractive index. This reduces the arrival time disparity because all modes arrive at about the same time. The modes traveling in a straight line are in a higher refractive index, so they travel slower than the serpentine modes. These travel farther but move faster in the lower refractive index of the outer core region.
V Parameter/V-Number:
The number of modes of multimode fiber cable depends on the wavelength of light, core diameter and material composition. This can be determined by the Normalized frequency parameter (V). The V is expressed as:

\[ V = \frac{\pi d}{\lambda} \sqrt{n_1^2 - n_2^2} = \frac{\pi d}{\lambda} (NA) \]

Where
d=fiber core diameter
\( \lambda \)=wavelength of light
NA=numerical aperture
For a single mode fiber, \( V \leq 2.405 \) and for multimode fiber, \( V \geq 20 \).
Mathematically, the number of modes for a step index, fiber is given by:

\[ N_{SI} = \frac{V^2}{2} \]

For a graded index fiber, the number of mode is given by:

\[ N_{GI} = \frac{V^2}{4} \]

Different type of dispersion in optical fiber:
Dispersion:
The dispersion is defined as the distortion of light wave or pulse as it travels from one end of the fiber to the other end of fiber. The data or information to be transmitted through fiber is first coded in the forms of pulse after these pulses are transmitted through the optical fiber. Finally, these pulses are received at the receiver and decoded. The light pulses, entering at different angles at input of fiber take different times to reach at the output end. Consequently the pulses are broaden at the output end. The pulses at input end, output ends are shown in

Fig. i.e

“The deformation in the pulse is called pulse dispersion.”

The pulse dispersion is of following types.
(1) Intermodal dispersion or modal dispersion
(2) Interamodal dispersion or chromatic dispersion
   a) Material dispersion b) Wave guide dispersion
(1) Intermodal dispersion or modal dispersion: Modal dispersion exists in multimode fibers. The mechanism of modal dispersion is, when light incident the fiber, it propagates in different mode. The higher order modes travel a long distance and arrive at the receiver end later than the lower order modes. In this way one mode travel more slowly than other. This shows that different modes have different group velocities.
(2) Material dispersion or spectral dispersion: This is wavelength based effect. Also we know the refractive index of core depends upon wavelength or frequency of light when a input pulse with different components travels with different velocities inside the fiber, the pulse broadens. This is known as material dispersion.

(3) Wave guide dispersion: Due to wave guide structure the light rays in the fiber follow different paths. Therefore they take different time interval to travel these path. This dispersion is called as waveguide dispersion.

**Signal losses in fiber communication.**

Signal Losses in optical Fiber are-

**Absorption Losses:** Absorption is the most prominent factor causing the attenuation in optical fiber. The absorption of light may be because of interaction of light with one or more major components of glass or caused by impurities within the glass. Following are the three main sources of absorption of light:

i) The absorption of light by the material of the core itself.

ii) The absorption of light by the presence of impurities in the fiber material.

iii) Loss of light energy because of atomic defects in the fiber material.

**Scattering losses:** During formation of optical fiber, sub-microscopic variation in the density and doping impurities are frozen into the glass. These becomes a source of reflection, refraction and scattering the light passing through the glass. Hence light is scattered in all direction and causes the loss of optical power. Such a loss of power is also known as Rayleigh scattering loss.

**Bending losses:** Bending losses occur due to imperfection and deformation present in the fiber structure. These are of two types:

- Micro bending losses: Micro bending losses occur due to defects in manufacturing process.
- Macro bending losses: Excessive bending of the fiber results the loss in light energy known as macro-bending loss. The macro-bending losses depends on the core radius and the bend radius.

**Attenuation in optical fiber communication.**

Attenuation:
Attenuation and pulse dispersion represent the two most important characteristics of an optical fiber that determine the information-carrying capacity of a fiber optic communication system. The decrease in signal strength along a fiber optic waveguide caused by absorption and scattering is known as attenuation. Attenuation is usually expressed in dB/km.

\[
Power\ loss(dB) = 10\log_{10}\frac{P_{out}}{P_{in}}
\]

\[
= 10\log_{10}k^L
\]

\[
= L.10\log_{10}k.
\]

\[
= \alpha. L
\]

where \(\alpha (= 10 \log_{10}k)\) is the attenuation coefficient of the fiber in dB/km.

Since attenuation is the loss, therefore, it is always expressed as

\[
P_{out} = P_{in}10^{-\frac{\alpha L}{10}}
\]

**Applications of optical fiber.**

The important applications of optical fiber are,

(1) Optical fibre communication has large bandwidth; it is capable of handling a number of channels. Hence it has wide applications in communication.

(2) The optical fibre system is used in defence services because high security is maintained.

(3) Optical fibre system are particularly suitable for transmission of digital data generated by computers.

(4) It is used for signaling purpose.

(5) Optical fibres are used in medical endoscopy.
A laser is a device that emits light through a process of optical amplification based on the stimulated emission of electromagnetic radiation. Laser is "light amplification by stimulated emission of radiation".

Laser Characteristics

- The light emitted from a laser has a very high degree of coherence. Where as the light emitted from conventional light source is incoherent because the radiation emitted from different atoms do not bear any definite phase relationship with each other.
- The light emitted from a laser is highly monochromatic.
- Degree of non-monochromaticity
- Lasers emit light that is highly directional, that is, laser light is emitted as a relatively narrow beam in a specific direction. Ordinary light, such as from a light bulb, is emitted in many directions away from the source.
- The intensity of Laser light is tremendously high as the energy is concentrated in a very narrow region and stays nearly constant with distance. The intensity of light from conventional source decreases rapidly with distance.

Einstein’s A,B Coefficients.

In 1916, Einstein considered the various transition rates between molecular states (say, 1 & 2)

a) Absorption

When an atom encounters a photon of light, it can absorb the photon’s energy and jump to an excited state.

Number of Absorption per unit time per unit volume = $B_{12} N_1 u(\nu)$

where $N_i$ is the number of atoms (per unit volume) in the $i^{th}$ state,

$U(\nu) \, d\nu$ radiation energy per unit volume within frequency range $\nu$ and $\nu+d\nu$

$B_{12}$ the coefficient of proportionality and is a characteristic of the energy levels

b) Spontaneous emission

Rate of Spontaneous emission (per unit volume) = $A_{21} N_2$

where $A_{21}$ is the proportionality constant
Molecules typically remain excited for no longer than a few nanoseconds.

c) Stimulated Emission

When a photon encounters an atom in an excited state, the photon can induce the atom to emit its energy as another photon of light, resulting in two photons.

\[
\text{Stimulated emission rate } = B_{21} N_2 u(\nu)
\]

Einstein first proposed stimulated emission in 1916.

**Relation between Einstein’s A, B Coefficient:** In thermal equilibrium, the rate of upward transitions equals the rate of downward transitions:

\[
B_{12} N_1 u(\nu) = A_{21} N_2 + B_{21} N_2 u(\nu)
\]

\[
u(\nu) = \frac{N_2 A_{21}}{N_1 B_{12} - N_2 B_{21}} = \frac{A_{21} / B_{21}}{B_{12} N_1^{-1} - 1} \quad \text{ ..........(1)}
\]

In equilibrium, the ratio of the populations of two states is given by the Maxwell–Boltzmann distribution

\[
N_2 / N_1 = \exp(-\Delta E/kT)
\]

where \( \Delta E = E_2 - E_1 = h\nu \), \( k \) is the Boltzmann Constant

As a result, higher-energy states are always less populated than the ground state

Acc. to **Planck’s blackbody radiation formula**
Common Components of all Lasers

1. **Active Medium**
   The active medium may be solid crystals such as ruby or Nd:YAG, liquid dyes, gases like CO₂ or Helium/Neon, or semiconductors such as GaAs. Active mediums contain atoms whose electrons may be excited to a metastable energy level by an energy source.

2. **Excitation Mechanism**
   Excitation mechanisms pump energy into the active medium by one or more of three basic methods; optical, electrical or chemical.

3. **High Reflectance Mirror**
   A mirror which reflects essentially 100% of the laser light.

4. **Partially Transmissive Mirror**
   A mirror which reflects less than 100% of the laser light and transmits the remainder.
Gas lasers consist of a gas filled tube placed in the laser cavity. A voltage (the external pump source) is applied to the tube to excite the atoms in the gas to a **population inversion**.

The number of atoms in any level at a given time is called the population of that level. Normally, when the material is not excited externally, the population of the lower level or ground state is greater than that of the upper level. When the population of the upper level exceeds that of the lower level, which is a reversal of the normal occupancy, the process is called **population inversion**. The light emitted from this type of laser is normally continuous wave (CW).

**Lasing Action**

1. Energy is applied to a medium raising electrons to an unstable energy level.
2. These atoms spontaneously decay to a relatively long-lived, lower energy, metastable state.
3. A population inversion is achieved when the majority of atoms have reached this metastable state.
4. Lasing action occurs when an electron spontaneously returns to its ground state and produces a photon.

1—pumping to raise atoms to an excited state
2—spontaneous emission emitting a photon
3—encounters with another excited atom causing stimulated emission
4—building up of more and more photons due to reflections
5—Laser beam
5. If the energy from this photon is of the precise wavelength, it will stimulate the production of another photon of the same wavelength and resulting in a cascading effect.

6. The highly reflective mirror and partially reflective mirror continue the reaction by directing photons back through the medium along the long axis of the laser.

7. The partially reflective mirror allows the transmission of a small amount of coherent radiation that we observe as the “beam”.

8. Laser radiation will continue as long as energy is applied to the lasing medium.

**Types of Laser**

**He-Ne laser**

A helium–neon laser or HeNe laser, is a type of gas laser whose gain medium consists of a mixture of helium and neon(10:1) inside of a small bore capillary tube, usually excited by a DC electrical discharge. The best-known and most widely used HeNe laser operates at a wavelength of 632.8 nm in the red part of the visible spectrum. The first He-Ne lasers emitted light at 1.15 μm, in the infrared spectrum, and were the first gas lasers.

The gain medium of the laser, is a mixture of helium and neon gases, in approximately a 10:1 ratio, contained at low pressure in a glass envelope. The gas mixture is mostly helium, so that helium atoms can be excited. The excited helium atoms collide with neon atoms, exciting some of them to the state that radiates 632.8 nm. Without helium, the neon atoms would be excited mostly to lower excited states responsible for non-laser lines. A neon laser with no helium can be constructed but it is much more difficult without this means of energy coupling. Therefore, a HeNe laser that has lost enough of its helium (e.g., due to diffusion through the seals or glass) will lose its laser functionality since the pumping efficiency will be too low. The energy or pump source of the laser is provided by a high voltage electrical discharge passed through the gas between electrodes (anode and cathode) within the tube. A DC current of 3 to 20 mA is typically required for CW operation. The optical cavity of the laser usually consists of two concave mirrors or one plane and one concave mirror, one having very high (typically 99.9%) reflectance and the output coupler mirror allowing approximately 1% transmission.
Commercial HeNe lasers are relatively small devices, among gas lasers, having cavity lengths usually ranging from 15 cm to 50 cm (but sometimes up to about 1 metre to achieve the highest powers), and optical output power levels ranging from 0.5 to 50 mW.

The mechanism producing population inversion and light amplification in a HeNe laser plasma originates with inelastic collision of energetic electrons with ground state helium atoms in the gas mixture. As shown in the accompanying energy level diagram, these collisions excite helium atoms from the ground state to higher energy excited states, among them the $2^3S_1$ and $2^1S_0$ long-lived metastable states. Because of a fortuitous near coincidence between the energy levels of the two He metastable states, and the $3s^2$ and $2s^2$ (Paschen notation) levels of neon, collisions between these helium metastable atoms and ground state neon atoms results in a selective and efficient transfer of excitation energy from the helium to neon.

**RUBY LASER**

- First laser to be operated successfully
- Lasing medium: Matrix of Aluminum oxide doped with chromium ions
- Energy levels of the chromium ions take part in lasing action
- A three level laser system
Working:

Ruby is pumped optically by an intense flash lamp. This causes Chromium ions to be excited by absorption of radiation around 0.55 µm and 0.40 µm.

Chromium ions are excited to levels $E_1$ and $E_2$

- Excited ions decay non-radiatively to the level $M$ – upper lasing level
- $M$- metastable level with a lifetime of ~ 3ms
- Laser emission occurs between level $M$ and ground state $G$ at an output wavelength of 0.6943 µm
- One of the important practical lasers
- Has long lifetime and narrow linewidth
- (Linewidth – width of the optical spectrum or width of the power Spectral density )
- Output lies in the visible region – where photographic emulsions and Photodetectors are much more sensitive
- than they are in infrared region
- Find applications in holography and laser ranging
  • Flash lamp operation – leads to a pulsed output of the laser
  • Between flashes, lasing action stops

**Laser spiking:** Output is highly irregular function of time

Intensity has random amplitude fluctuations of varying duration

**Applications of laser**

1. **Scientific**
   - a. Spectroscopy
   - c. Photochemistry
   - e. Nuclear fusion

2. **Military**
   - a. Death ray
   - b. Defensive applications
c. Strategic defense initiative  
d. Laser sight  
e. Illuminator  
f. Target designator